Semiconductor qubits for quantum computation

Is it possible to realize a quantum computer with semiconductor technology?

Matthias Fehr
TU Munich
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St.Petersburg/Russia

Contents

The history of quantum computation

1936 Alan Turing

- Church-Turing thesis:

  There is a „Universal Turing machine“, that can efficiently simulate any other algorithm on any physical device

1982 Feynman

- Computer based on quantum mechanics might avoid problems in simulating quantum mech. systems

1985 Deutsch

- Search for a computational device to simulate an arbitrary physical system

  quantum mechanics -> quantum computer

  Efficient solution of algorithms on a quantum computer with no efficient solution on a Turing machine?

The history of quantum computation

1994 Peter Shor	• Efficient quantum algorithms - prime factorization - discrete logarithm problem ->more power
1995 Lov Grover	• Efficient quantum search algorithm
In the 1990s	• Efficient simulation of quantum mechanical systems
1995 Schumacher	• “Quantum bit” or “qubit” as physical resource
1996 Calderbank, Shor, Steane	• Quantum error correction codes - protecting quantum states against noise

The basics of quantum computation

- Classical bit: 0 or 1
- 2 possible values

- Qubit: $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$
- α, β are complex -> infinite possible values -> continuum of states

Qubit measurement: result 0 with probability $|\alpha|^2$

result 1 with probability $|\beta|^2$

$$|\alpha|^2 + |\beta|^2 = 1$$

Wave function collapses during measurement,

qubit will remain in the measured state

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \xrightarrow{|\alpha|^2} |0\rangle \xrightarrow{100\%} |0\rangle$$

Qubits

<u>Bloch sphere:</u>

We can rewrite our state with phase factors γ, θ, φ

$$|\Psi\rangle = e^{i\gamma}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right)$$

Qubit realizations: 2 level systems

1) ground- and excited states of electron orbits
2) photon polarizations
3) alignment of nuclear spin in magnetic field
4) electron spin

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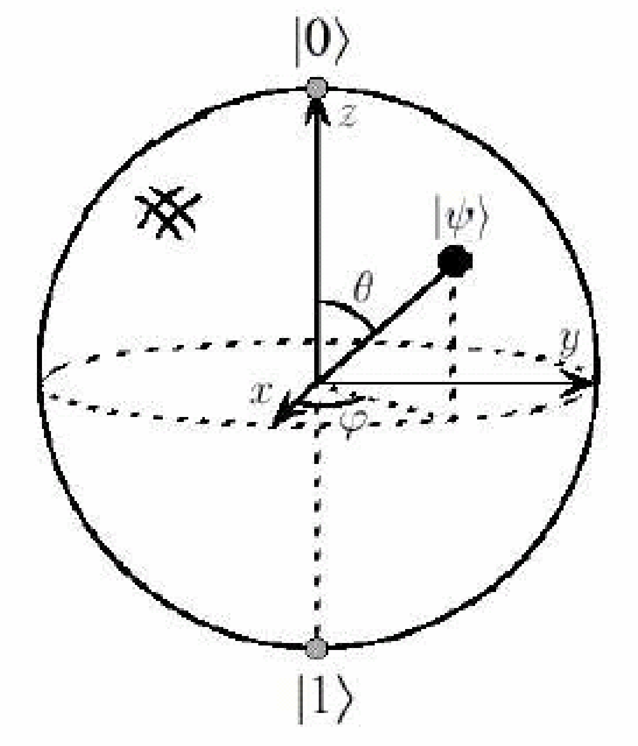


Bloch sphere [from Nielson&Chuang]

Single qubit gates

- Qubits are a possibility to store information quantum mechanically
- Now we need operations to perform calculations with qubits
- -> quantum gates:

- NOT gate: X

  classical NOT gate: 0 -> 1; 1 -> 0

  quantum NOT gate: $\alpha|0\rangle + \beta|1\rangle \Rightarrow \alpha|1\rangle + \beta|0\rangle$

- Linear mapping -> matrix operation X

  Equal to the Pauli spin-matrix

$$X \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \sigma_x$$

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$X \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

Single qubit gates

- Every single qubit operation can be written as a matrix U
- Due to the normalization condition every gate operation U has to be unitary
- -> Every unitary matrix specifies a valid quantum gate
- Only 1 classical gate on 1 bit, but ∞ quantum gates on 1 qubit.
- Z-Gate leaves $|0\rangle$ unchanged, and flips the sign of $|1\rangle \rightarrow -|1\rangle$
- Hadamard gate = “square root of NOT”

$$UU^* = I$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Hadamard gate

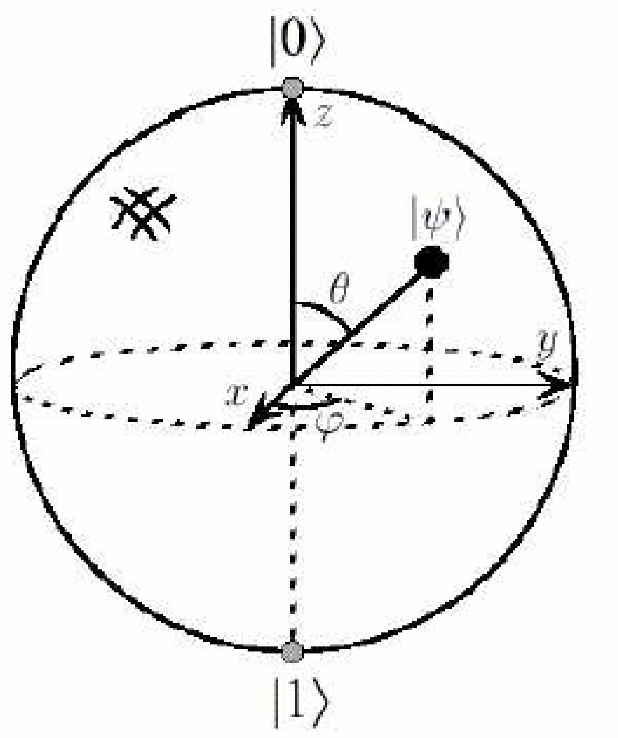
- <u>Bloch sphere:</u>
  - Rotation about the y-axis by 90°
  - Reflection through the x-y-plane

$$\alpha|0\rangle + \beta|1\rangle \xrightarrow{H} \alpha\frac{|0\rangle+|1\rangle}{\sqrt{2}} + \beta\frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

- Creating a superposition

$$|0\rangle \xrightarrow{H} \frac{|0\rangle+|1\rangle}{\sqrt{2}}$$

$$|1\rangle \xrightarrow{H} \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$



Bloch sphere [from Nielson&Chuang]

Decomposing single qubit operations

- An arbitrary unitary matrix can be decomposed as a product of rotations

$$U = e^{i\alpha} \begin{bmatrix} e^{-i\beta/2} & 0 \\ 0 & e^{i\beta/2} \end{bmatrix} \begin{bmatrix} \cos\gamma/2 & -\sin\gamma/2 \\ \sin\gamma/2 & \cos\gamma/2 \end{bmatrix} \begin{bmatrix} e^{-i\delta/2} & 0 \\ 0 & e^{i\delta/2} \end{bmatrix}$$

- 1^{st} and 3^{rd} matrix: rotations about the z-axis

- 2^{nd} matrix: normal rotation

- Arbitrary single qubit operations with a *finite* set of quantum gates

- Universal gates

Multiple qubits

For quantum computation multiple qubits are needed!

<u>2 qubit system:</u>

computational bases stats:

superposition:

1st qubit 2nd qubit

$$|00\rangle, |01\rangle, |10\rangle, |11\rangle$$

$$|\Psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

<u>Measuring a subset of the qubits:</u>

Measurement of the 1st qubit gives 0 with probability $|\alpha_{00}|^2 + |\alpha_{01}|^2$

leaving the state

$$|\Psi`\rangle = \frac{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}$$

Entanglement

- <u>Bell state or EPR pair:</u>

  prepare a state: $\left|\Psi\right\rangle = \frac{\left|00\right\rangle + \left|11\right\rangle}{\sqrt{2}}$

- Measuring the 1st qubit gives

  0 with prop. 50% leaving $\left|\Psi\right\rangle = \left|00\right\rangle$

  1 with prop. 50% leaving $\left|\Psi\right\rangle = \left|11\right\rangle$

- The measurement of the 2nd qubits always gives the same result as the first qubit!
- The measurement outcomes are correlated!

- Non-locality of quantum mechanics
- Entanglement means that state can not be written as a product state

$$\left|\Psi\right\rangle = \left|\Psi_1\right\rangle\left|\Psi_2\right\rangle = \left(\frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}}\right)\left(\frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}}\right) =$$

$$\frac{\left|0\right\rangle\left|0\right\rangle + \left|0\right\rangle\left|1\right\rangle + \left|1\right\rangle\left|0\right\rangle + \left|1\right\rangle\left|1\right\rangle}{2} \neq \frac{\left|00\right\rangle + \left|11\right\rangle}{\sqrt{2}}$$

Multiple qubit gates, CNOT

- Classical: AND, OR, XOR, NAND, NOR -> NAND is universal
- Quantum gates: NOT, CNOT

$$U_{CN}\left|\Psi\right\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{bmatrix}$$

- <u>CNOT gate:</u>
  - controlled NOT gate = classical XOR
  - If the control qubit is set to 0, target qubit is the same
  - If the control qubit is set to 1, target qubit is flipped

  $$\left|00\right\rangle \to \left|00\right\rangle; \left|01\right\rangle \to \left|01\right\rangle; \left|10\right\rangle \to \left|11\right\rangle; \left|11\right\rangle \to \left|10\right\rangle$$

  $$\left|A,B\right\rangle \to \left|A, B \oplus A\right\rangle \qquad \oplus \equiv \text{mod}\, 2$$

- <u>CNOT is *universal* for quantum computation</u>
- Any multiple qubit logic gate may be composed from CNOT and single qubit gates

- Unitary operations are reversible
  (unitary matrices are invertible, U unitary -> U^{-1} too)
- Quantum gate are always reversible, classical gates are not reversible

Qubit copying?

- classical: CNOT copies bits
- Quantum mech.: *impossible*

- We try to copy an unknown state $|\Psi\rangle = a|0\rangle + b|1\rangle$
- Target qubit: $|0\rangle$
- Full state: $\left[a|0\rangle + b|1\rangle\right]|0\rangle = a|00\rangle + b|10\rangle$

- Application of CNOT gate: $a|00\rangle + b|11\rangle = |\Psi\rangle|\Psi\rangle$
- We have successful copied $|\Psi\rangle$, but only in the case $|\Psi\rangle = |0\rangle\ or\ |1\rangle$

- General state $|\Psi\rangle|\Psi\rangle = a^2|00\rangle + ab|01\rangle + ab|10\rangle + b^2|11\rangle$

- *No-cloning* theorem: major difference between quantum and classical information

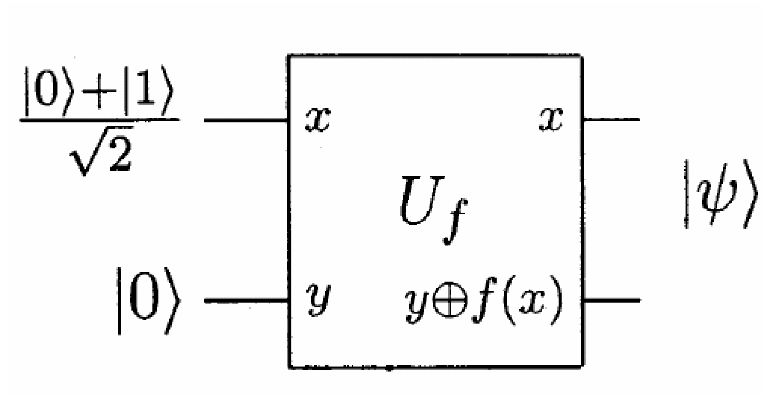
Quantum parallelism

- Evaluation of a function: $f(x):\{0,1\}\rightarrow\{0,1\}$
- Unitary map: black box

$$U_f : |x, y\rangle \rightarrow |x, y \oplus f(x)\rangle$$

- Resulting state:

$$|\Psi\rangle = \frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$



Quantum gate [from Nielson&Chuang (2)]

- Information on f(0) and f(1) with a single operation

$$|0, f(0)\rangle$$

$$|1, f(1)\rangle$$

- Not immediately useful, because during measurement the superposition will collapse

Deutsch algorithm

- Input state: $|\Psi_0\rangle = |01\rangle$

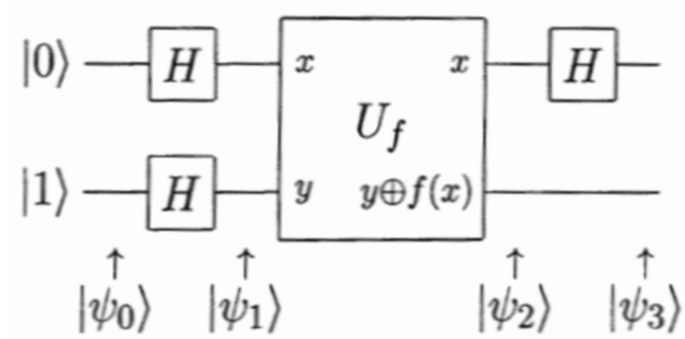
$$|\Psi_1\rangle = \left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] = |x\rangle\big(|0\rangle - |1\rangle\big)/\sqrt{2}$$

- Application of U_f :

$$|\Psi_2\rangle = (-1)^{f(x)}|x\rangle\big(|0\rangle - |1\rangle\big)/\sqrt{2}$$

$$|\psi_2\rangle = \pm\left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right]\left[\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right] \quad \text{if} \quad f(0) = f(1)$$

$$|\psi_2\rangle = \pm\left[\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right]\left[\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right] \quad \text{if} \quad f(0) \neq f(1)$$



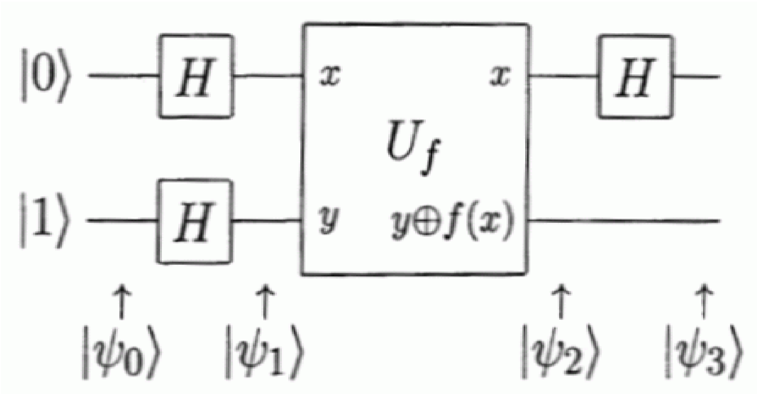
$$|\psi_3\rangle = \pm|0\rangle\left[\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right] \quad \text{if} \quad f(0) = f(1)$$

$$|\psi 3\rangle = \pm|1\rangle\left[\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right] \quad \text{if} \quad f(0) \neq f(1)$$

Deutsch algorithm

$$|\Psi_3\rangle = \pm |f(0)\oplus f(1)\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$

- global property determined with one evaluation of f(x)
- classically: 2 evaluations needed
- Faster than any classical device
- Classically 2 alternatives exclude one another
- In quantum mech.: *interference*



$$f(0)\oplus f(1) = 0 \;\; if \;\; f(0) = f(1)$$

$$f(0)\oplus f(1) = 1 \;\; if \;\; f(0) \neq f(1)$$

$$|\psi_3\rangle = \pm|0\rangle\left[\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right] \quad \text{if} \quad f(0) = f(1)$$

$$|\psi 3\rangle = \pm|1\rangle\left[\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right] \quad \text{if} \quad f(0) \neq f(1)$$

Quantum algorithms

	Classical steps	quantum logic steps
Fourier transform e.g.: - Shor's prime factorization - discrete logarithm problem - Deutsch Jozsa algorithm	$N\log(N) = n2^n$ $N = 2^n$ - n qubits - N numbers	$\log^2(N) = n^2$ - hidden information! - Wave function collapse prevents us from directly accessing the information
Search algorithms	N	$\sqrt{N}$
Quantum simulation	c^N bits	kn qubits

The Five Commandments of DiVincenzo

1. A physical system containing qubits is needed

2. The ability to initialize the qubit state
   $\left|000...\right\rangle$

3. Long decoherence times, longer than the gate operation time
   - Decoherence time: 10^4-10^5 x “clock time”
   - Then error-correction can be successful

4. A universal set of quantum gates (CNOT)

5. Qubit read-out measurement

Realization of a quantum computer

- Systems have to be almost completely isolated from their environment
- The coherent quantum state has to be preserved
- Completely preventing decoherence is impossible
- Due to the discovery of quantum error-correcting codes, slight decoherence is tolerable
- Decoherence times have to be very long -> implementation realizable
- Performing operations on several qubits in parallel
- 2- Level system as qubit:
  - Spin ½ particles
  - Nuclear spins
- Read-out:
  - Measuring the single spin states
  - Bulk spin resonance

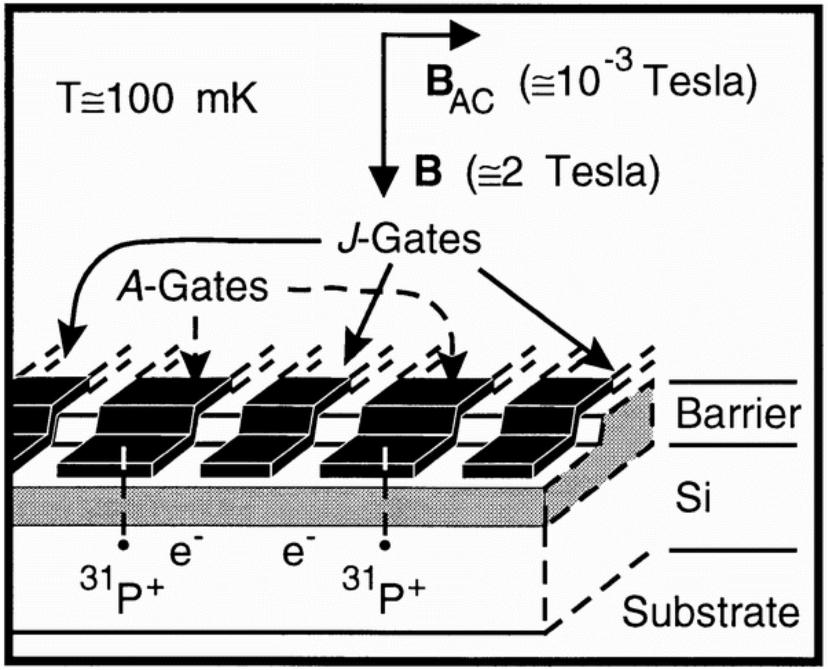
Si:^{31}P, Kane concept from 1998

- Logical operations on *nuclear spins* of ^{31}P(I=1/2) donors in a Si host(I=0)

- Weakly bound ^{31}P valence electron at T=100mK

- Spin degeneracy is broken by B-field
- Electrons will only occupy the lowest energy state when $2\mu_B B >> kT$
- Spin polarization by a strong B-field and low temperature

- Long ^{31}P spin relaxation time $\approx 10^{18}\,s$, due to low T



Single spin rotations

- Hyperfine interaction $A \propto |\Psi|^2$ at the nucleus

$$H_{en} = \mu_B B \sigma_z^e - g_n \mu_n B \sigma_z^n + A \sigma^e \cdot \sigma^n$$

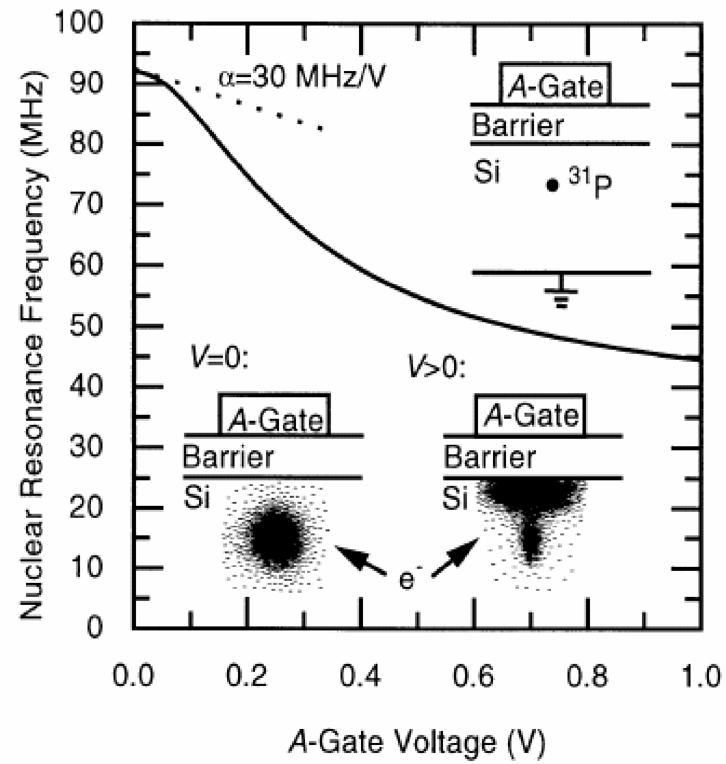
$$h\nu_A = 2 g_n \mu_n B + 2A + \frac{2A^2}{\mu_B B}$$

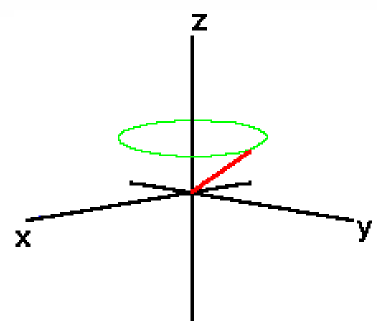
- ν_A : frequency separation of the nuclear levels

- A-gate voltage pulls the electron wave function envelope away from the donors
- *Precession frequency* of nuclear spins is controllable

- 2nd magnetic field B_{ac} in resonance to the changed precession frequency
- Selectively addressing qubits
- Arbitrary spin rotations on each nuclear spin

$$\phi = \Delta g_{eff} \, \mu_B \, \tau \, B_{ac} / 2\hbar$$





Qubit coupling

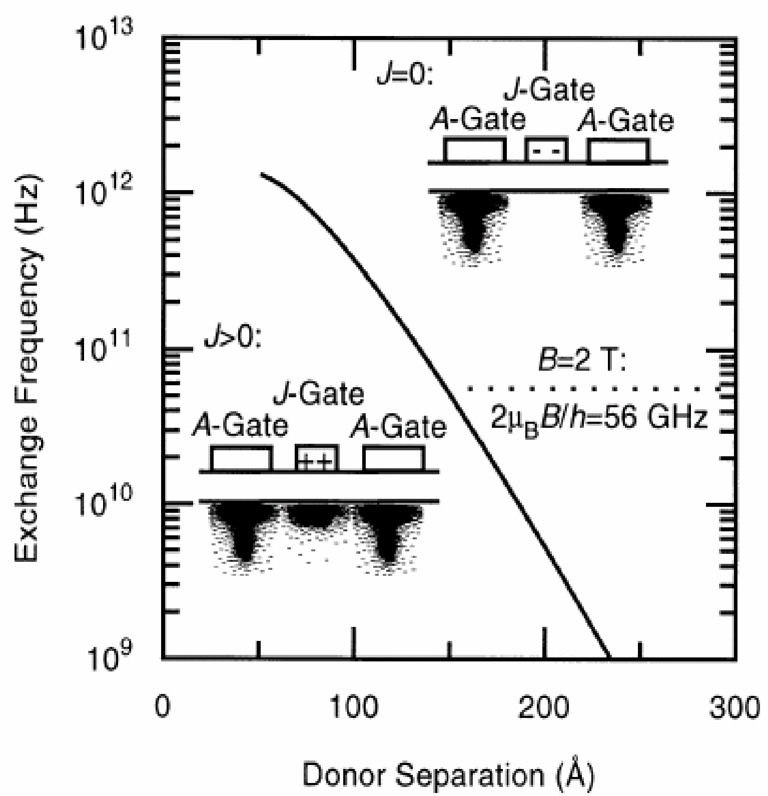
- J-gates influence the neighboring electrons -> qubit coupling

- Strength of exchange coupling depends on the overlap of the wave function

$$H = H(B) + A_1\sigma^{1n}\cdot\sigma^{2e} + A_2\sigma^{2n}\cdot\sigma^{2e} + J\sigma^{1e}\cdot\sigma^{2e}$$

$$4J(r) \cong 1.6\frac{e^2}{\varepsilon a_B}\left(\frac{r}{a_B}\right)^{5/2}\exp\left(\frac{-2r}{a_B}\right)$$

- Donor separation: 100-200 $\mathring{A}$
- Electrons mediate nuclear spin interactions, and facilitate measurement of nuclear spins



Qubit measurement

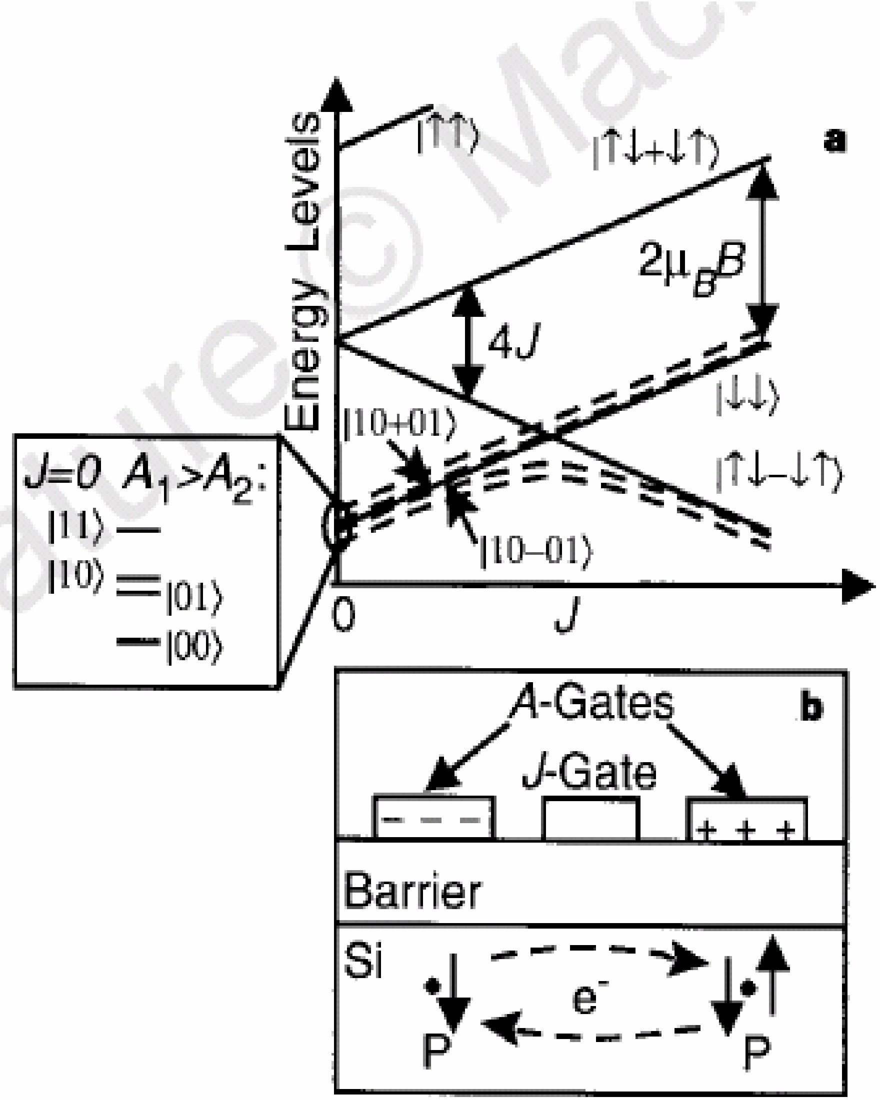
- $J < \mu_B B/2$: qubit operation
- $J > \mu_B B/2$: qubit measurement

- Orientation of nuclear spin 1 alone determines if the system evolves into singlet or triplet state

- Both electrons bound to same donor (D^- state, singlet)

- Charge motion between donors
- Single-electron capacitance measurement

- Particles are indistinguishable



Many problems

- Materials free of spin($I \neq 0$ isotopes)
- Ordered 1D or 2D-donor array
- Single atom doping methodes
- Grow high-quality Si layers on array surface
- 100-A-scale gate devices
- Every transistor is individual -> large scale calibration
- A-gate voltage increases the electron-tunneling probability
- Problems with low temperature environment
  - Dissipation through gate biasing
  - Eddy currents by B_{ac}
  - Spins not fully polarized

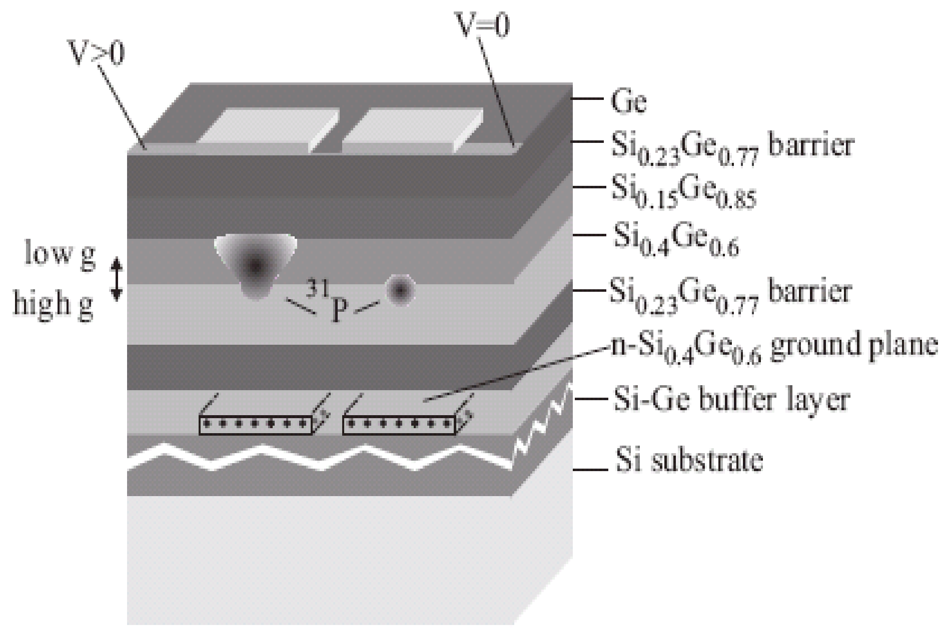
SRT with Si-Ge heterostructures

- Spin resonance transistors, at a size of 2000 A
- Larger Bohr radius (larger m^*, ε)
- Done by electron beam lithography

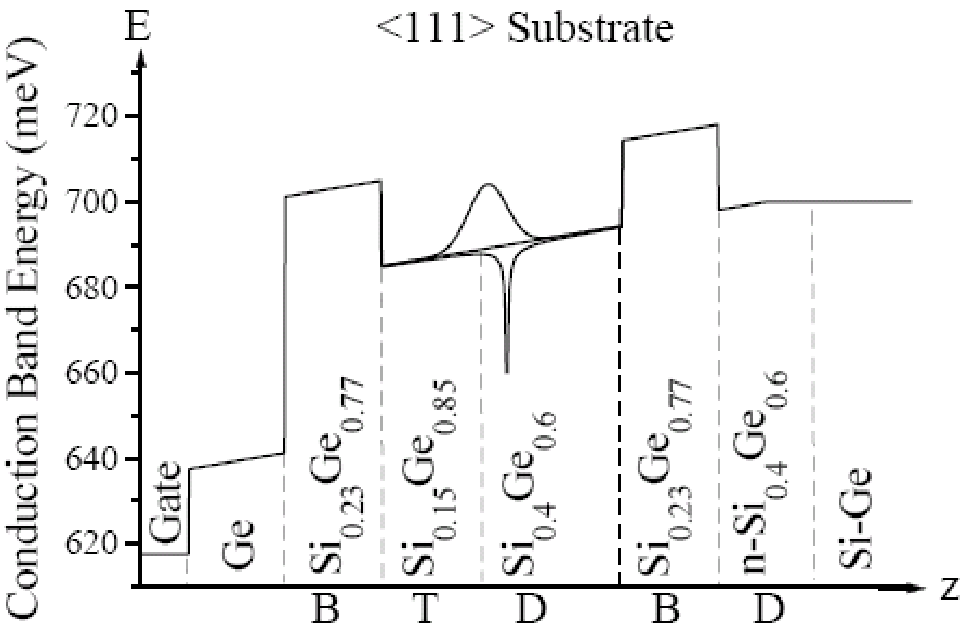
- Electron spin as qubit
  - Isotropic purity not critical
  - No needed spin transfer between nucleus and electrons

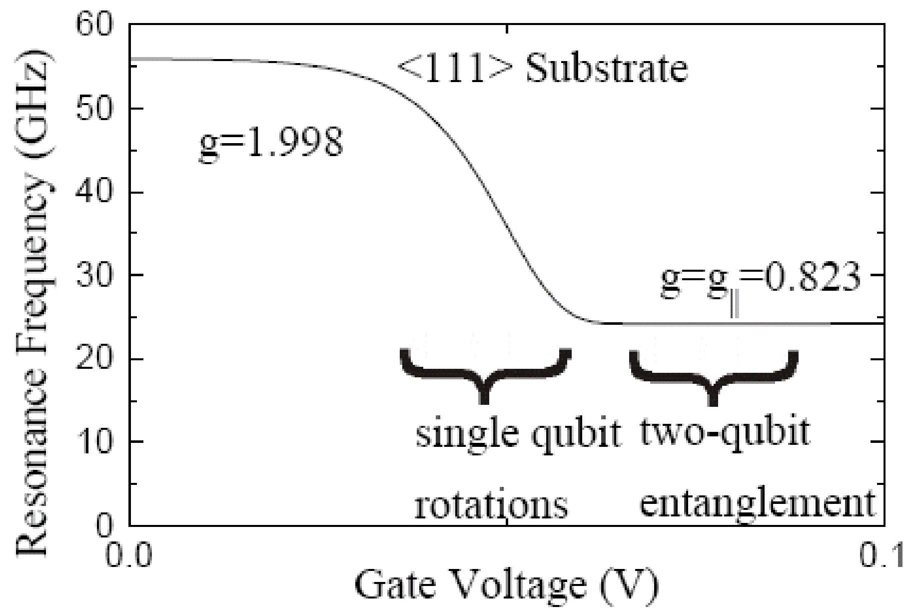
- *Different g-factors*

  Si: g=1.998 / Ge: g=1.563
- Spin Zeeman energy changes
- Gate bias pulls wave function away from donor



Confinement and spin rotations

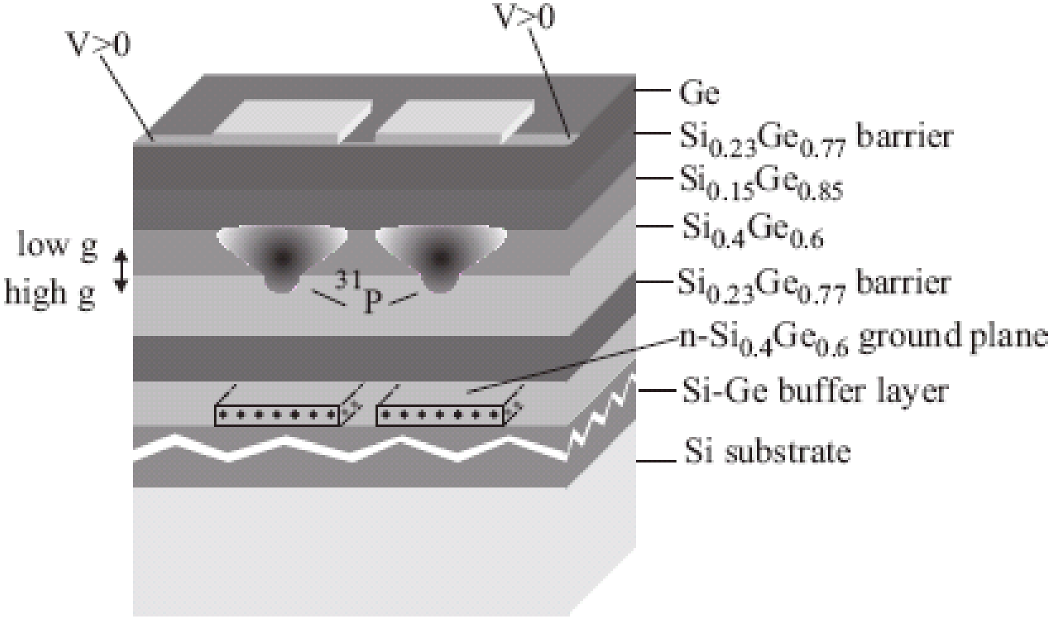




- Confinement through B-layer
- RF-field in resonance with SRT -> arbitrary spin phase change

2-qubit interaction

- No J-gate needed
- Both wave functions are pulled near the B-layer
- Coulomb potential weakens
- Larger Bohr radius
- Overlap can be tuned
- CNOT gate

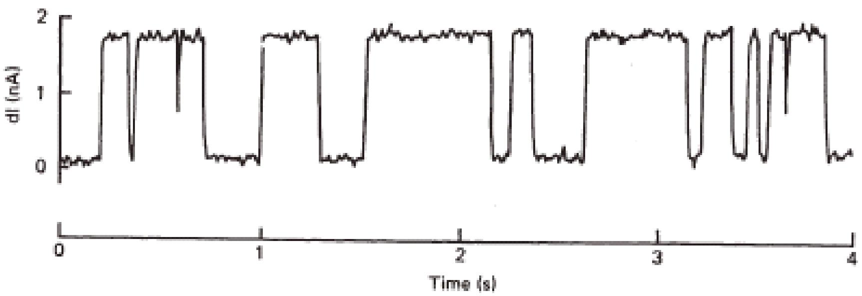


Detection of spin resonance

- <u>FET channel:</u>
  n-$Si_{0.4}Ge_{0.6}$ ground plane counter-electrode

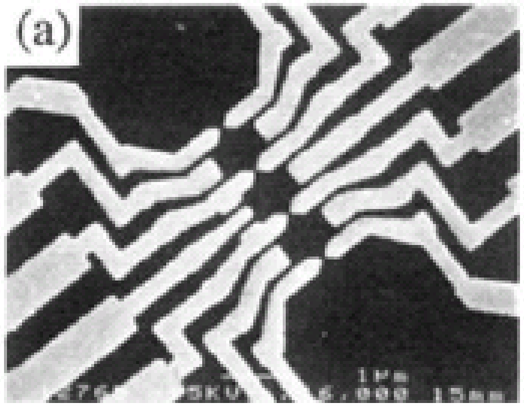
- Qubit between FET channel and gate electrode
- Channel current is sensitive to donor charge states:
  - ionized / neutral / doubly occupied (D^- state)

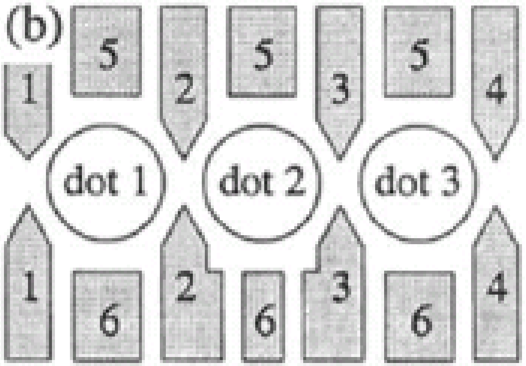
- D^- state (D^+ state) on neighbor transistors, change in channel current -> Singlet state
- Channel current constant -> triplet state

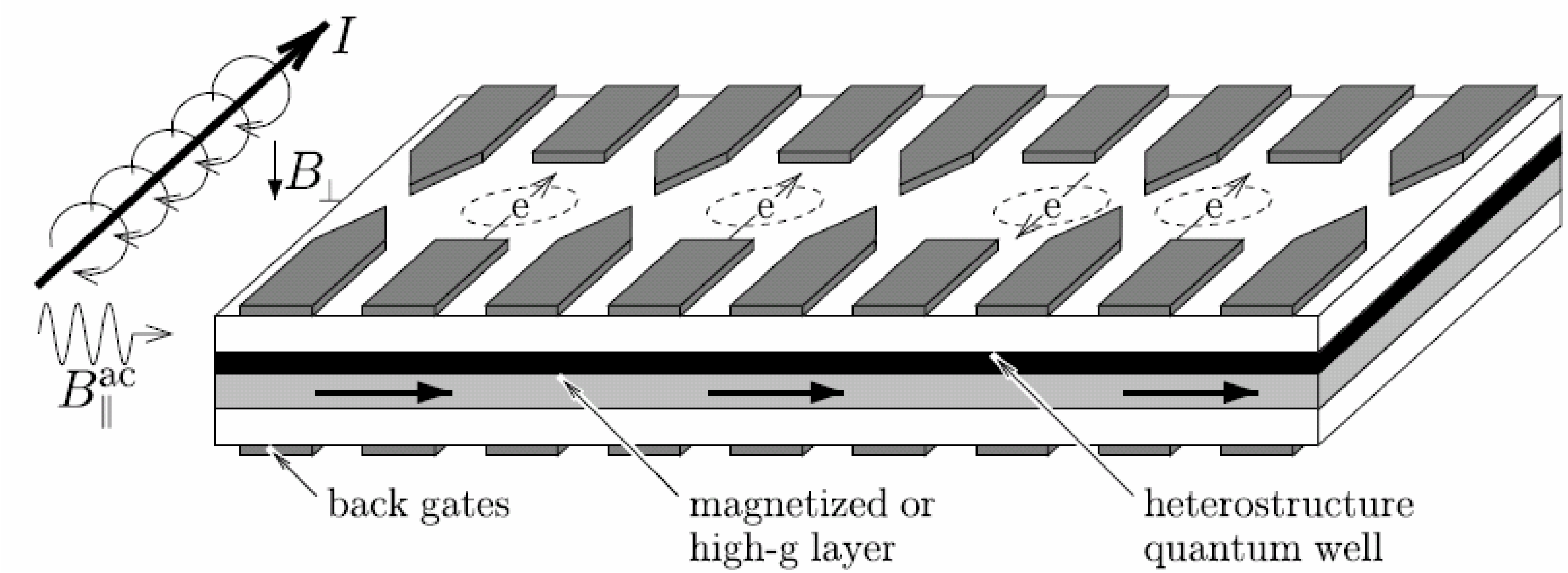


Electro-statically defined QD

- GaAs/AlGaAs heterostructure -> 2DEG
- address qubits with
  - high-g layer
  - gradient B-field
- Qubit coupling by lowering the tunnel barrier







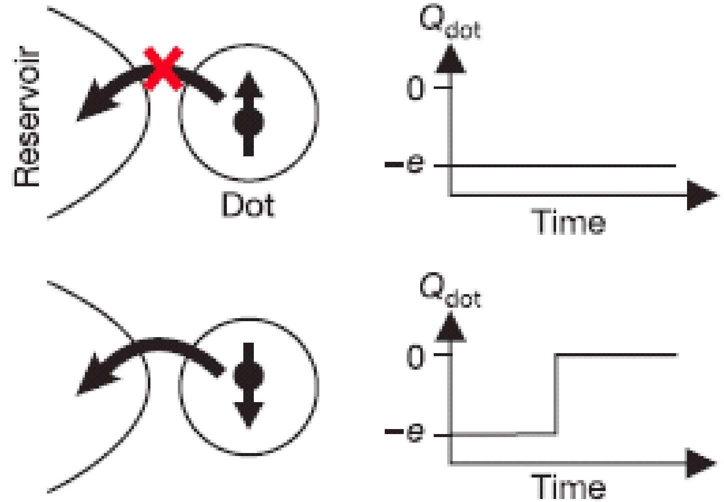
Single spin read-out in QD

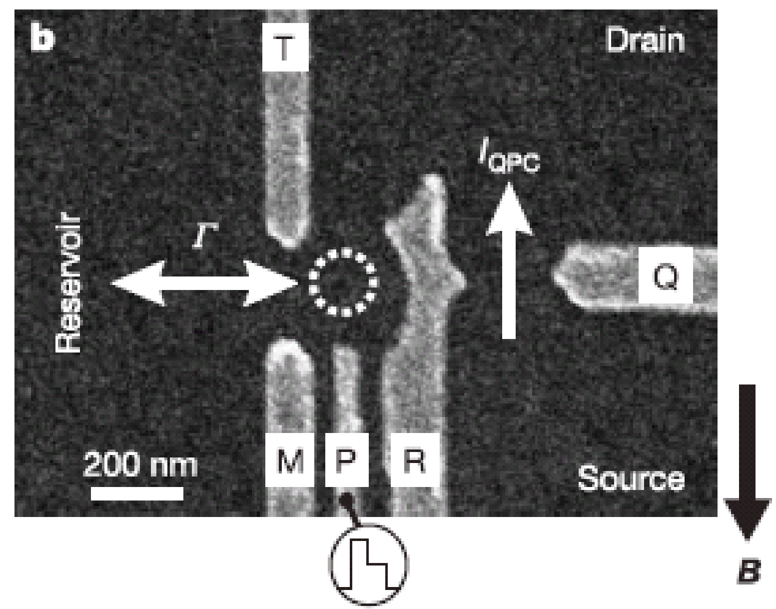
- Spin-to-charge conversion of electron confined in QD (circle)
- Magnetic field to split states

- GaAs/AlGaAs heterostructure -> 2DEG
- Dots defined by gates M, R, T
- Potential minimum at the center

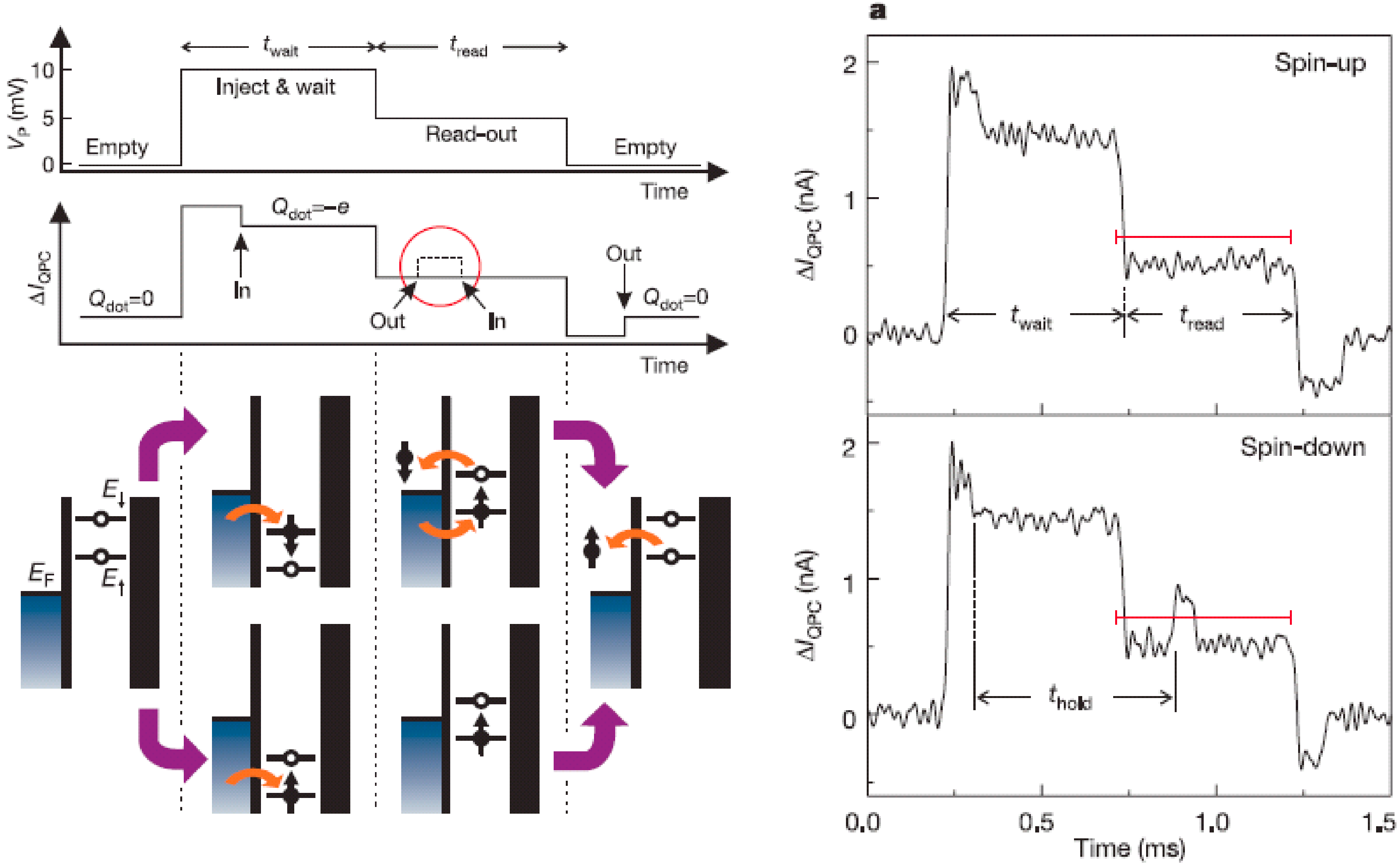
- Electron will leave when spin-↓
- Electron will stay when spin-↑
- QPC as charge detector

- Electron tunneling between reservoir and dot
- Changes in Q_{QPC} detected by measuring I_{QPC}





Two-level pulse on P-gate

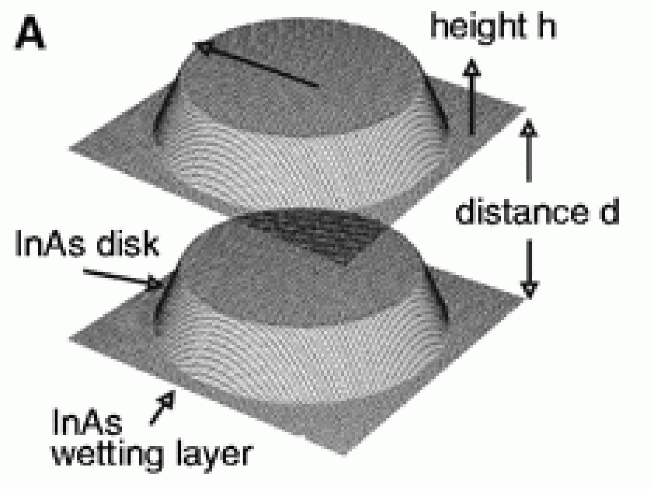


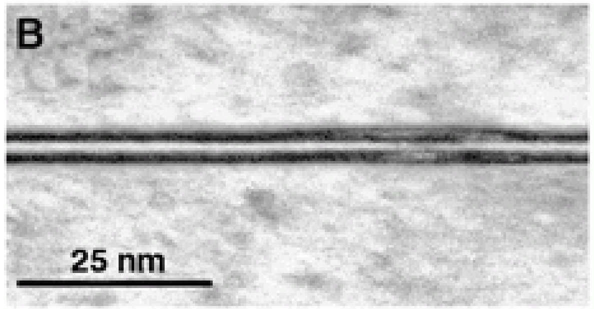
Self-assembled QD-molecule

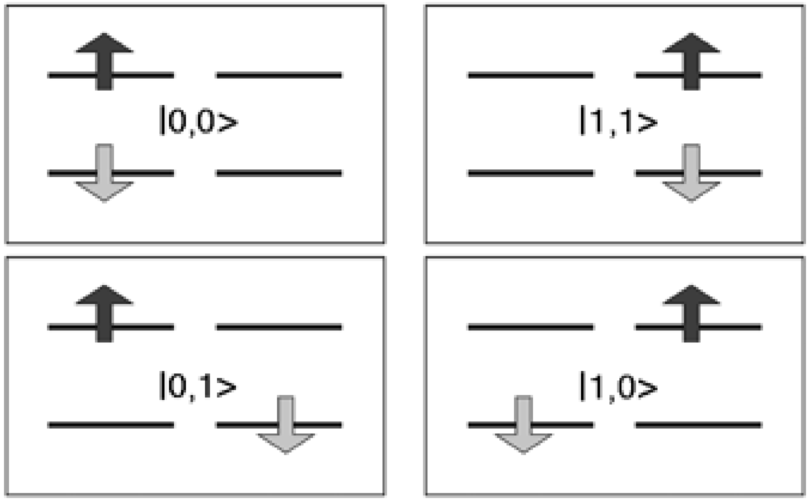
- Coupled InAs quantum dots
  - quantum molecule

- Vertical electric field localizes carriers
- Upper dot = index 0
- Lower dot = index 1

- Optical created exciton
- Electric field off -> tunneling -> entangled state

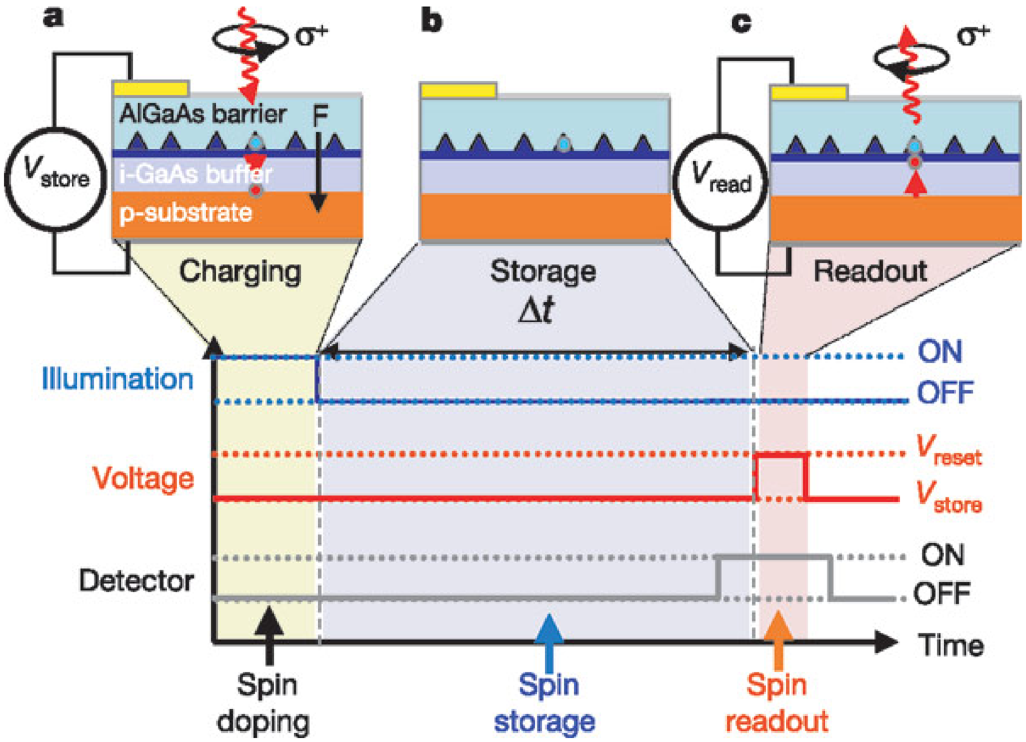






Self-assembled Quantum Dots array

- Single QD layer
- Optical resonant excitation of e-h pairs
- Electric field forces the holes into the GaAs buffer
- Single electrons in the QD ground state (remains for hours, at low T)
- V_{read}: holes drift back and recombine
- Large B-field: Zeeman splitting of exciton levels



Self-assembled Quantum Dots array

- Circularly polarized photons

$$\sigma^+ \rightarrow +1\hbar$$

$$\sigma^- \rightarrow -1\hbar$$

$$J_z = J_{e,z} + J_{h,z} = \pm 1\hbar$$

$$e\uparrow h\downarrow \; and \; e\downarrow h\uparrow$$

- Mixed states
- Zeeman splitting yields either

$$e\uparrow h\downarrow \; or \; e\downarrow h\uparrow$$

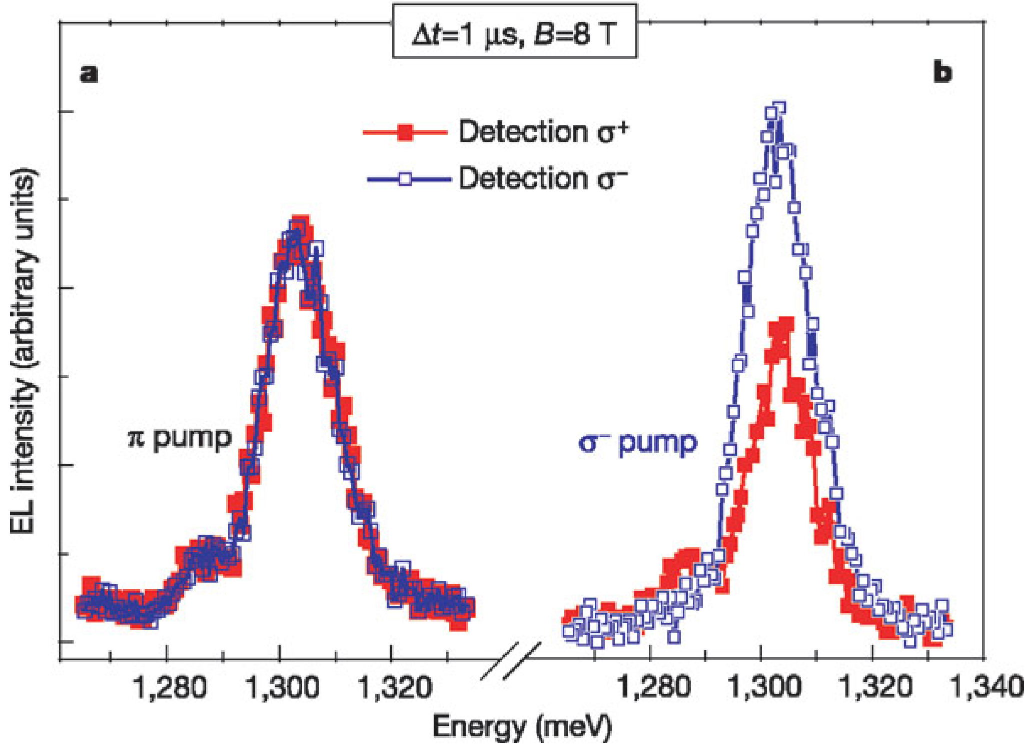
- Optical selection of pure spin states

$$e\uparrow h\downarrow \Rightarrow J = -1\hbar$$

$$e\downarrow h\uparrow \Rightarrow J = +1\hbar$$

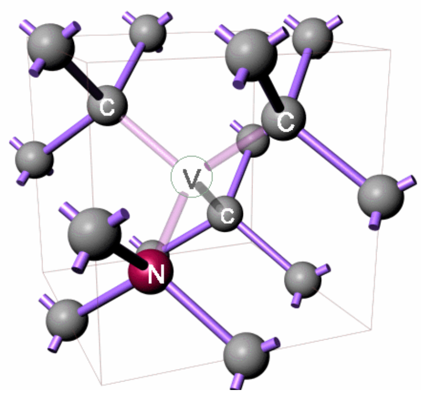
$$J_{e,z} = +1/2\hbar \quad e\uparrow \quad J_{h,z} = +3/2\hbar \quad h\uparrow$$

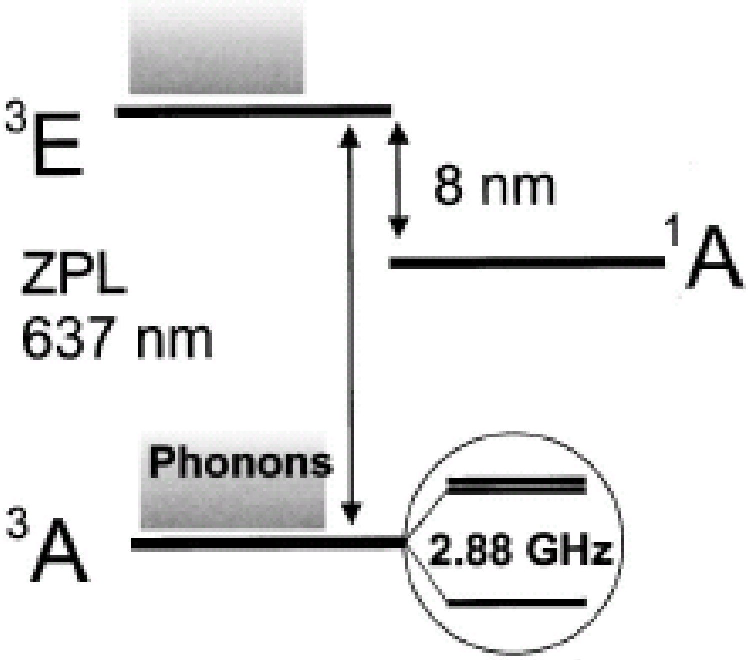
$$J_{e,z} = -1/2\hbar \quad e\downarrow \quad J_{h,z} = -3/2\hbar \quad h\downarrow$$



NV$^-$ center in diamond

- Nitrogen Vacancy center: defect in diamond, N-impurity
- ^3A -> ^3E transition: spin conserving
- ^3E -> ^1A transition: spin flip
- Spin polarization of the ground state
- Axial symmetry -> ground state splitting at zero field
- B-field for Zeeman splitting of triplet ground state
- Low temperature spectroscopy





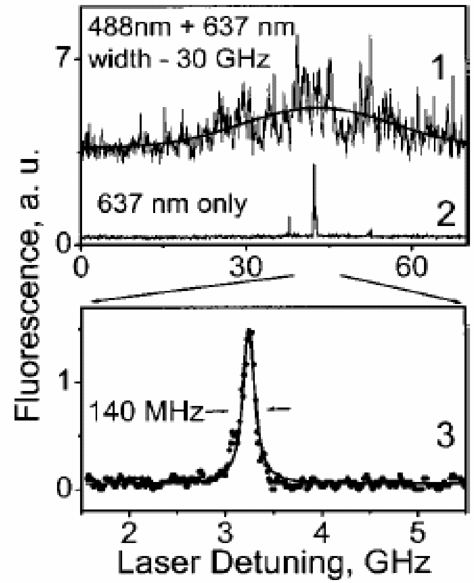
NV$^-$ center in diamond

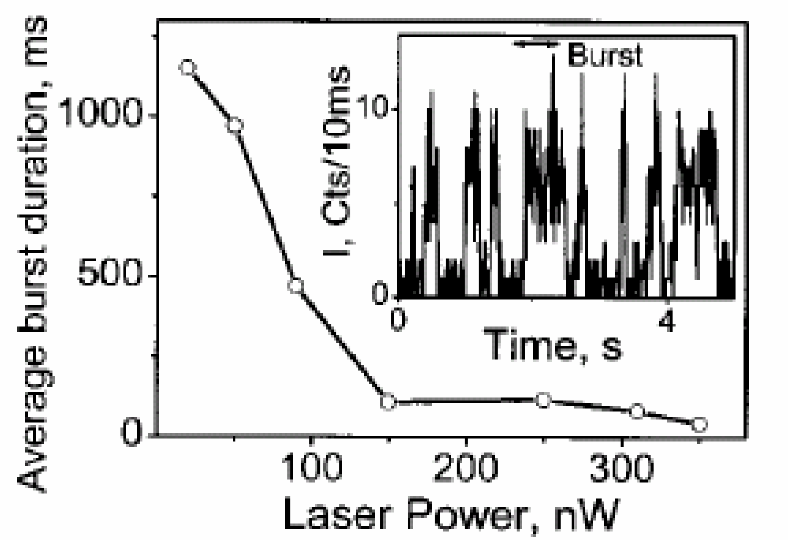
- Fluorescence excitation with laser

- Ground state energy splitting greater than transition line with
- Excitation line marks spin configuration of defect center

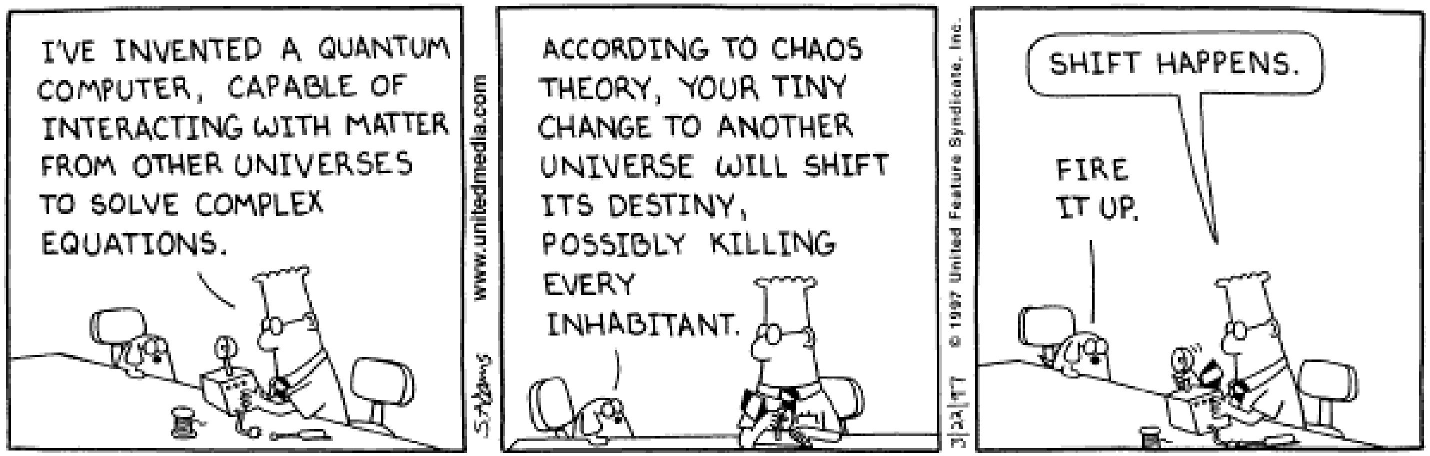
- <u>On resonant excitation:</u>
- Excitation-emission cycles ^3A -> ^3E
  - bright intervals, bursts

- Crossing to ^1A singlet small
  - No resonance
  - Dark intervals in fluorescence





Thank you very much!



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