Modeling of contact interaction of two bodies with possibility of sliding

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Content

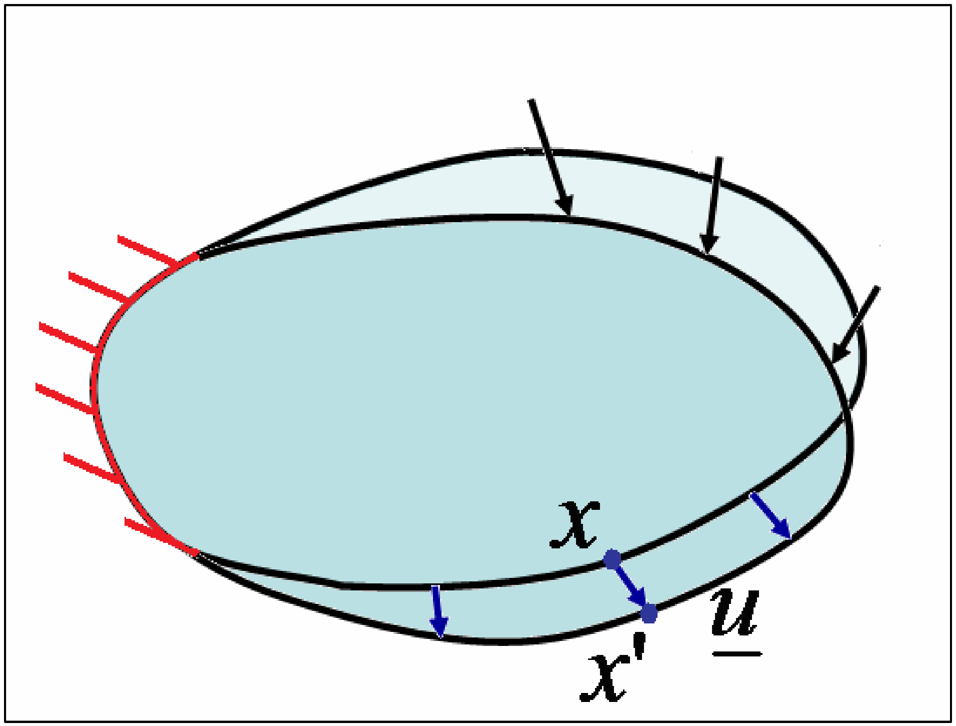
Theoretical part

- Classical statement of contact problems
- Integrated statement
- Variation statement
- Solvability of contact problems on basis of minimization of convex functional

Practical part

- Description of fast algorithm for solving contact problems
- Investigation of applicability of the algorithm to models with great curvature of surfaces
- Analysis of influence of tangential displacements to the solution
- Comparing the results obtained with developed algorithm and finite element complex ANSYS

Problem 1. Deformation of elastic body without obstacle



- Small deformations in the body:

$$\underline{\underline{\varepsilon}} = \frac{\nabla \underline{u} + \nabla^T \underline{u}}{2}$$

- Hook's law:

$$\underline{\underline{\sigma}} = \underline{\underline{L}} : \underline{\underline{\varepsilon}}$$

- Balance equations:

$$\mathrm{div}\left(\underline{\underline{\sigma}}\left(\underline{u}\right)\right) = -\underline{K}$$

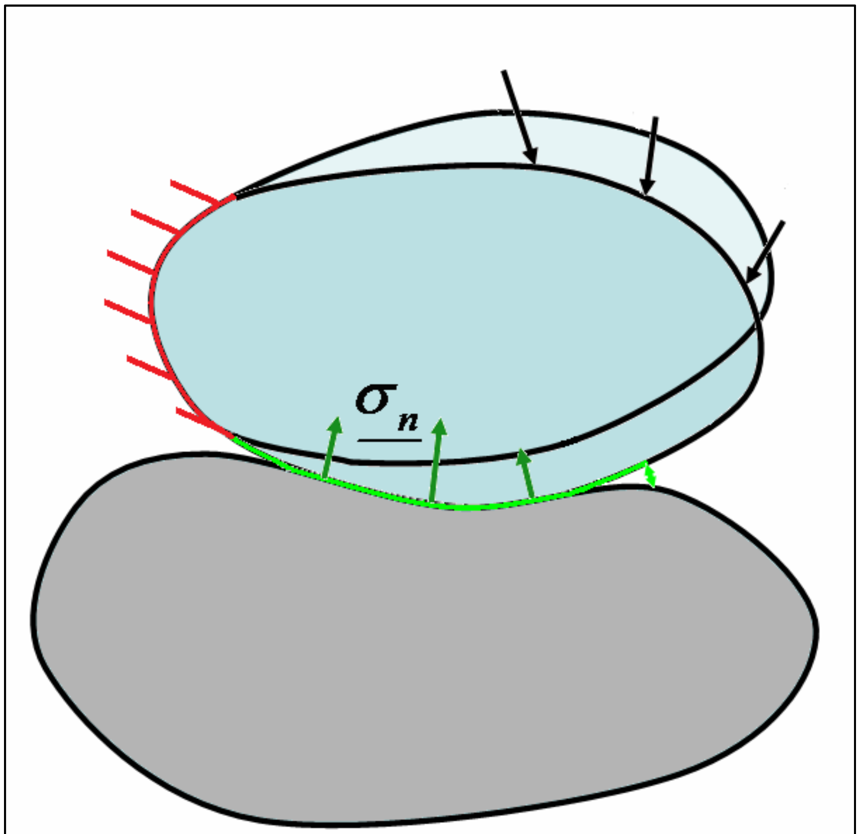
- Boundary conditions:

$$\underline{u}\Big|_{\partial_1\Omega} = 0$$

$$\underline{\sigma_n}\Big|_{\partial_2\Omega} = \underline{F_n}$$

- Solution: $\underline{u} \in \left[C^1\left(\overline{\Omega}\right) \cap C^2(\Omega)\right]^n$

Problem 2. Contact of elastic body and rigid obstacle



- Balance equations:

$$\operatorname{div}\left(\underline{\underline{\sigma}}(\underline{u})\right) = -\underline{K}$$

- Boundary conditions:

$$\underline{u}\big|_{\partial_1\Omega} = 0$$

$$\underline{\sigma_n}\big|_{\partial_2\Omega} = \underline{F_n}$$

- Condition of non-penetration:

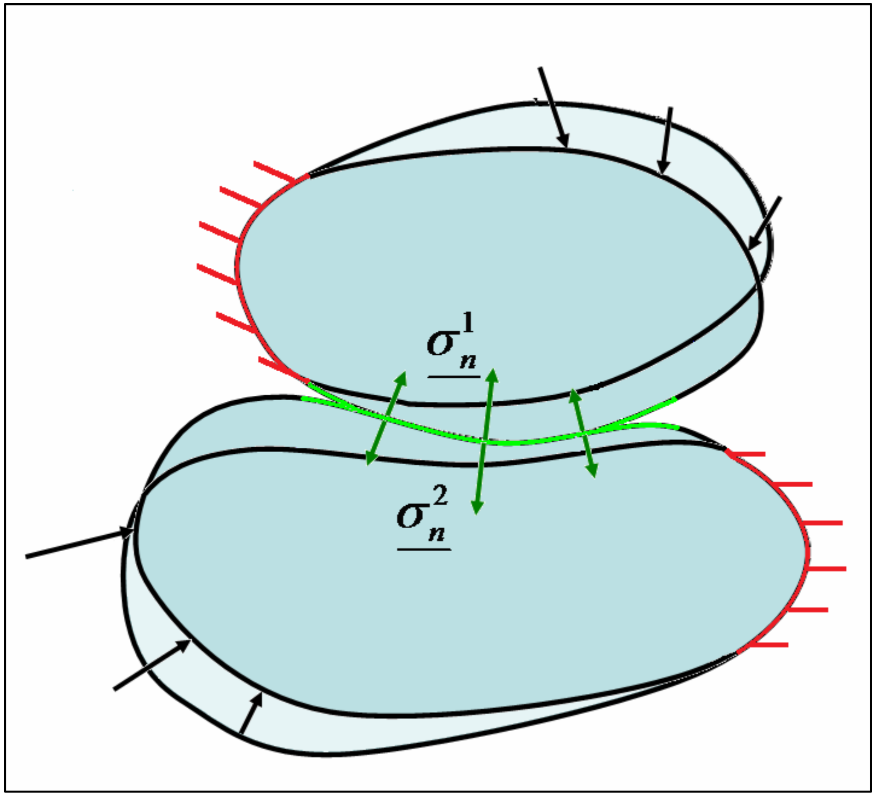
$$u_n\big|_{\partial_3\Omega} \le g_n$$

- Signorini conditions:

$$\begin{cases} \underline{\sigma_n}\big|_{\partial_3\Omega} = \sigma_n \underline{n}, \quad \sigma_n \le 0; \\ \sigma_n = 0, \quad \text{если} \quad u_n < g_n \end{cases}$$

- Solution: $\underline{u} \in \left[C^1\left(\overline{\Omega}\right) \cap C^2(\Omega)\right]^n$

Problem 3. Contact of two rigid bodies



- Balance equations:

$$\operatorname{div}\left(\underline{\underline{\sigma}}(\underline{u})\right) = -\underline{K}$$

- Boundary conditions:

$$\underline{u}\big|_{\partial_1\Omega} = 0$$

$$\underline{\sigma_n}\big|_{\partial_2\Omega} = \underline{F_n}$$

- Conditions in contact area:

$$u_n\big|_{\partial_3\Omega_1} + u_n\big|_{\partial_3\Omega_2} \le g_n$$

$$\begin{cases} \underline{\sigma_n}\big|_{\partial_3\Omega} = \sigma_n \underline{n}, \quad \sigma_n \le 0; \\ \sigma_n = 0, \text{ если } u_n\big|_{\partial_3\Omega_1} + u_n\big|_{\partial_3\Omega_2} < g_n; \\ \sigma_n\big|_{\partial_3\Omega_1} = \sigma_n\big|_{\partial_3\Omega_2} \end{cases}$$

- Solution: $\underline{u} \in \left[C^1\left(\overline{\Omega}\right) \cap C^2(\Omega)\right]^n$

Problem 3. Integrated statement

- Classical statement
- Integrated statement

$$\underline{u} \in \left[C^1(\overline{\Omega}) \cap C^2(\Omega)\right]^n$$

$$\begin{cases} \operatorname{div}(\underline{\underline{\sigma}}(\underline{u})) = -\underline{K} \\ \underline{u}\big|_{\partial_1\Omega} = 0 \\ \underline{\sigma_n}\big|_{\partial_2\Omega} = \underline{F_n} \\ \underline{\sigma_n}\big|_{\partial_3\Omega} = \sigma_n \underline{n}, \quad \sigma_n \le 0 \\ \sigma_n = 0, \text{ если } u_n\big|_{\partial_3\Omega_1} + u_n\big|_{\partial_3\Omega_2} < g_n \\ \sigma_n\big|_{\partial_3\Omega_1} = \sigma_n\big|_{\partial_3\Omega_2} \end{cases}$$

$$\underline{u} \in \Im,$$

$$\Im = \left\{ \begin{array}{l} \underline{f} \in \left[W_2^1(\Omega)\right]^n ; \quad \underline{f}\big|_{\partial_1\Omega} = 0; \\ f_n\big|_{\partial_3\Omega_1} + f_n\big|_{\partial_3\Omega_2} \le g_n \end{array} \right\}$$

$$\int_\Omega \underline{\underline{L}} : \underline{\underline{\varepsilon}}(\underline{u}) : \underline{\underline{\varepsilon}}(\underline{v} - \underline{u})\, dx \ge \int_\Omega \underline{K} \cdot (\underline{v} - \underline{u})\, dx + \int_{\partial_2\Omega} \underline{F_n} \cdot (\underline{v} - \underline{u})\, ds,$$

$$\forall\, \underline{v} \in \Im$$

Integrated statements of contact problems

- Problem 1 $\quad \underline{u} \in \left[W_2^1(\Omega)\right]^n \qquad \hat{W}_2^1(\Omega) = \left\{ w \in W_2^1(\Omega); \quad w\big|_{\partial_1\Omega} = 0 \right\}$

$$\int_\Omega \underline{\underline{L}} : \underline{\underline{\varepsilon}}(\underline{u}) : \underline{\underline{\varepsilon}}(\underline{v})\, dx = \int_\Omega \underline{K} \cdot \underline{v}\, dx + \int_{\partial_2\Omega} \underline{F_n} \cdot \underline{v}\, ds, \quad \forall\, \underline{v} \in \left[W_2^1(\Omega)\right]^n \quad \longrightarrow \quad \boxed{a(\underline{u}, \underline{v}) = l(\underline{v})}$$

- Problem 2 $\quad \underline{u} \in \Re \qquad \Re = \left\{ \underline{w} \in \left[W_2^1(\Omega)\right]^n; \ \underline{w}\big|_{\partial_1\Omega} = 0; \ w_n\big|_{\partial_3\Omega} \le g_n \right\}$

$$\int_\Omega \underline{\underline{L}} : \underline{\underline{\varepsilon}}(\underline{u}) : \underline{\underline{\varepsilon}}(\underline{v} - \underline{u})\, dx \ge \int_\Omega \underline{K} \cdot (\underline{v} - \underline{u})\, dx + \int_{\partial_2\Omega} \underline{F_n} \cdot (\underline{v} - \underline{u})\, ds, \quad \forall\, \underline{v} \in \Re \quad \longrightarrow \quad \boxed{a(\underline{u}, \underline{v} - \underline{u}) \ge l(\underline{v} - \underline{u})}$$

- Problem 3 $\quad \underline{u} \in \Im \qquad \Im = \left\{ \underline{w} \in \left[W_2^1(\Omega)\right]^n; \ \underline{w}\big|_{\partial_1\Omega} = 0; \ w_n^1\big|_{\partial_3\Omega_1} + w_n^2\big|_{\partial_3\Omega_2} \le g_n \right\}$

$$\int_\Omega \underline{\underline{L}} : \underline{\underline{\varepsilon}}(\underline{u}) : \underline{\underline{\varepsilon}}(\underline{v} - \underline{u})\, dx \ge \int_\Omega \underline{K} \cdot (\underline{v} - \underline{u})\, dx + \int_{\partial_2\Omega} \underline{F_n} \cdot (\underline{v} - \underline{u})\, ds, \quad \forall\, \underline{v} \in \Im \quad \longrightarrow \quad \boxed{a(\underline{u}, \underline{v} - \underline{u}) \ge l(\underline{v} - \underline{u})}$$

Minimization of functional

- Problem 1
- Problem 2
- Problem 3

$$a(\underline{u},\underline{v}) = l(\underline{v}), \forall \underline{v} \in \left[W_2^1(\Omega)\right]^n$$

$$a(\underline{u},\underline{v}-\underline{u}) \geq l(\underline{v}-\underline{u}), \forall \underline{v} \in \Re$$

$$a(\underline{u},\underline{v}-\underline{u}) \geq l(\underline{v}-\underline{u}), \forall \underline{v} \in \Im$$

$$\min_{\underline{v}\in A} I(\underline{v}), \quad I(\underline{v}) = \frac{1}{2} a(\underline{v},\underline{v}) - l(\underline{v})$$

Set A

$$\left(\left[W_2^1(\Omega)\right]^n,\ \Re,\ \Im\right)$$

- Self-contained
- Convex

Functional I

- Continuous
- Strict convex
- Bottom bounded
- Coercive

$$\exists!\underline{u} \in A,$$
$$I(\underline{u}) = \min_{\underline{v}\in A} I(\underline{v})$$

Algorithm for solving contact problems

Variation statement → Finite-dimensional model → Numeric algorithm

- Data preparation
- Computation

Computation the gap between the bodies

Input: rigidity matrix of the system K

vector of initial gap g_n

vector of applied loads F

Solving minimization problem:

obtain the vector of displacements U, which brings the minimum to the functional I,

$$I(U) = U^T \cdot K \cdot U - 2 \cdot F^T \cdot U$$

with the restrictions $U \le g_n$

Output: gap field in the contact area $G = g_n - U$

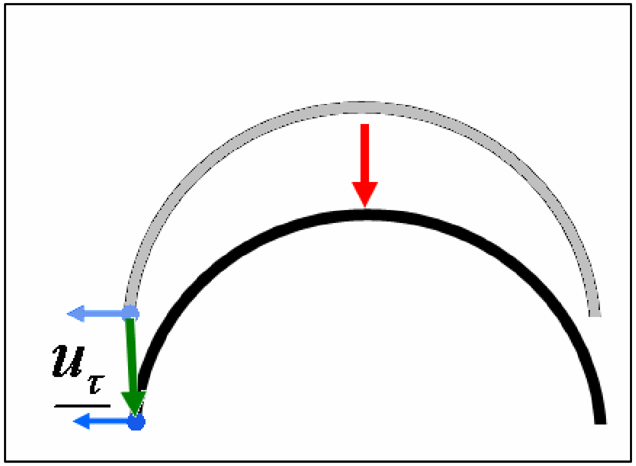
Problems of applicability to the models with great curvature

Algorithm features

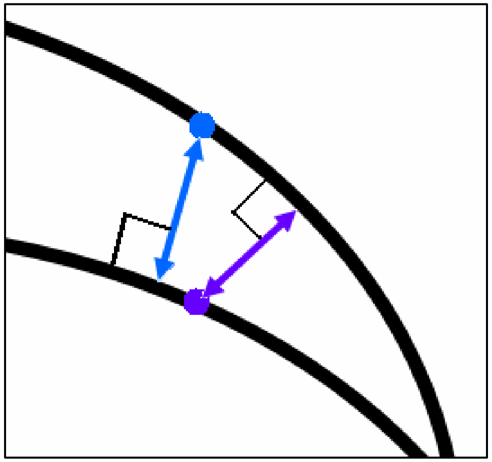
- Modeling of junction of the models with slight curvature of surfaces
- Applying loads in normal direction
- Computing displacements in normal directions

Difficulties in junction of the models with great curvature

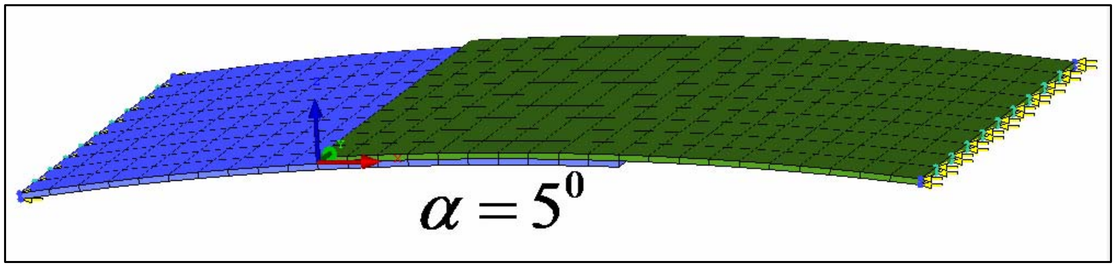
- Great tangential displacements

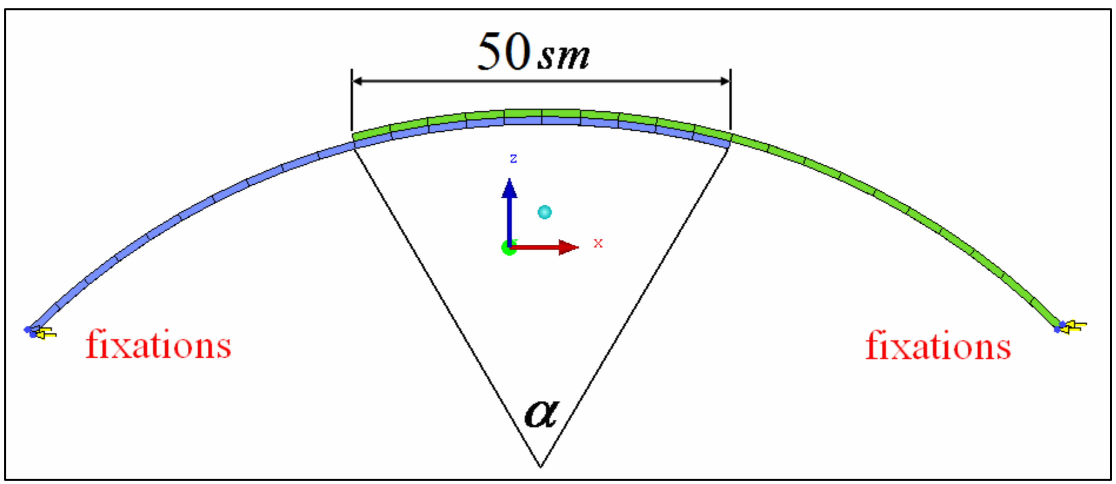


- Ambiguity in gap definition

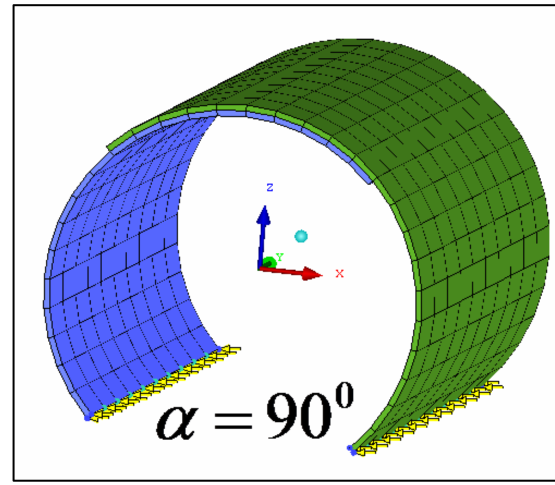


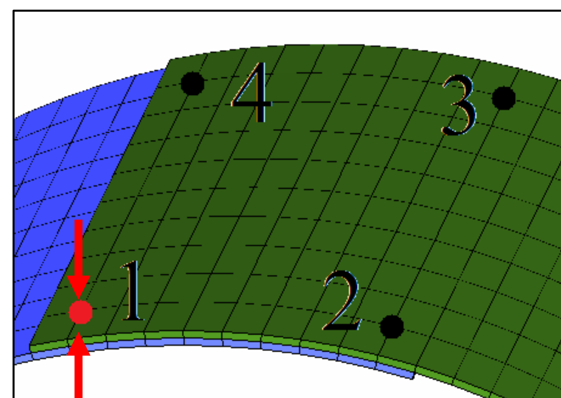
Test model description





$$G_{unif} = 7\ мм$$





Tangential displacements in gap calculation

Value of tangential displacements

$$rel = \frac{u_\tau}{u_n} 100\%$$

Difference in solutions with and without including tangential displacements

$$\varepsilon = \frac{\left| G^n - G^{xyz} \right|}{G_{unif}} 100\%$$

$\alpha = 5^0$

$F = 5\,kg$

1	3.29
2	3.62
3	3.47
4	3.55

$\alpha = 5^0$

$F = 3\,kg$

1	0.09
2	0.81
3	0.39
4	0.13

$\alpha = 90^0$

$F = 5\,kg$

1	36.94
2	150.45
3	63.86
4	87.07

$\alpha = 90^0$

$F = 17\,kg$

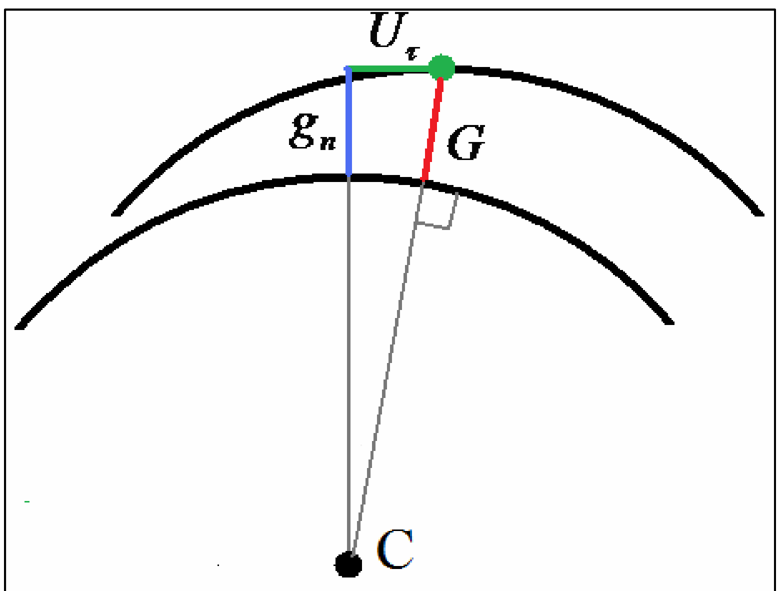
1	0.13
2	0.27
3	0.10
4	0.26

Comparing the methods of gap definition

- As the difference between initial gap and normal displacements:

$$G = g_n - U$$

- According to the model geometry:



Difference in solutions with the two methods

$$\varepsilon = \frac{\left|G^{dif} - G^{geom}\right|}{G_{unif}} 100\%$$

$\alpha = 5^0$

$F = 22\,kg$

1	0.00
2	0.01
3	0.00
4	0.01

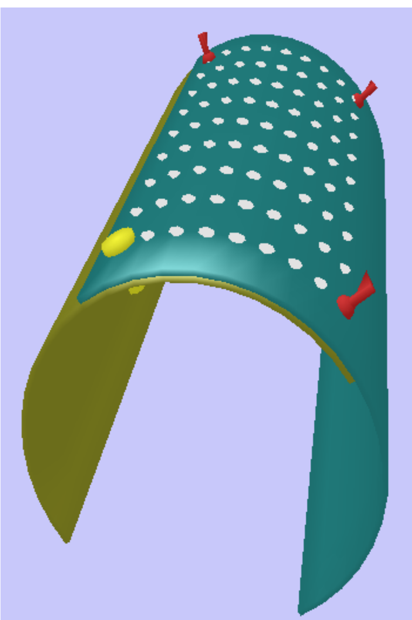
$\alpha = 90^0$

$F = 50\,kg$

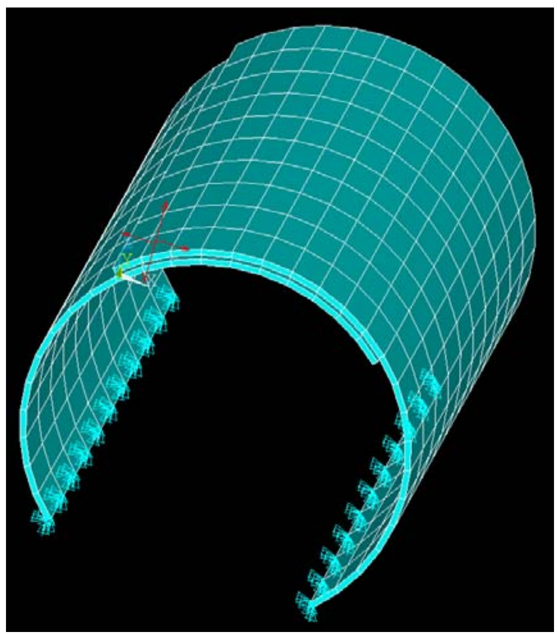
1	0.86
2	0.34
3	0.41
4	0.80

Comparing results of solution with ANSYS

- Developed algorithm



- ANSYS



Difference in solutions with the two algorithms

$$\varepsilon = \frac{\left|G^{ANSYS} - G\right|}{G_{unif}} 100\%$$

$$\alpha = 5^0$$

$$F = 5\,kg$$

1	0.70
2	4.91
3	3.31
4	1.39

$$\alpha = 90^0$$

$$F = 29\,kg$$

1	2.33
2	2.90
3	1.60
4	4.19

Conclusion

- Mathematical statements of contact problems without friction were investigated
- Solvability of these contact problems was proved on basis of minimization of convex functional

- Applicability of developed fast algorithm for solving contact problems to modeling of junction of models with great curvature was investigated
- Good correspondence of the results obtained with developed algorithm and finite element complex ANSYS was demonstrated
- Tangential displacements don’t influence on the gap value, what allows to apply the developed algorithm to modeling of junction of models with great curvature

Thank you!