17 Gomory Hu Trees

Given an undirected, weighted graph $G = (V, E, c)$ a cut-tree $T = (V, F, w)$ is a tree with edge-set F and capacities w that fulfills the following properties.

1. **Equivalent Flow Tree:** For any pair of vertices $s, t \in V$, $f(s,t)$ in G is equal to $f_T(s,t)$.
2. **Cut Property:** A minimum s-t cut in T is also a minimum cut in G.

Here, $f(s,t)$ is the value of a maximum s-t flow in G, and $f_T(s,t)$ is the corresponding value in T.

Overview of the Algorithm

The algorithm maintains a partition of V, (sets $S_1, \dots, S_t$), and a spanning tree T on the vertex set $\{S_1, \dots, S_t\}$.

Initially, there exists only the set $S_1 = V$.

Then the algorithm performs $n - 1$ split-operations:

- In each such split-operation it chooses a set S_i with $|S_i| \geq 2$ and splits this set into two non-empty parts X and Y.
- S_i is then removed from T and replaced by X and Y.
- X and Y are connected by an edge, and the edges that before the split were incident to S_i are attached to either X or Y.

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Details of the Split-operation

- Select S_i that contains at least two nodes a and b.
- Compute the connected components of the forest obtained from the current tree T after deleting S_i. Each of these components corresponds to a set of vertices from V.
- Consider the graph H obtained from G by contracting these connected components into single nodes.
- Compute a minimum a-b cut in H. Let A, and B denote the two sides of this cut.
- Split S_i in T into two sets/nodes $S_i^a \coloneqq S_i \cap A$ and $S_i^b \coloneqq S_i \cap B$ and add edge $\{S_i^a, S_i^b\}$ with capacity $f_H(a,b)$.
- Replace an edge $\{S_i, S_x\}$ by $\{S_i^a, S_x\}$ if $S_x \subset A$ and by $\{S_i^b, S_x\}$ if $S_x \subset B$.

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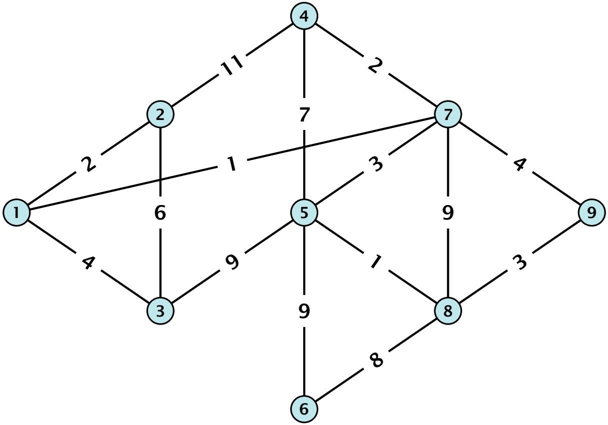

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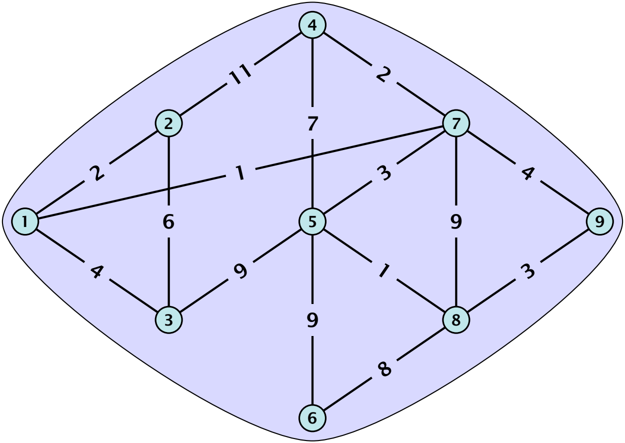

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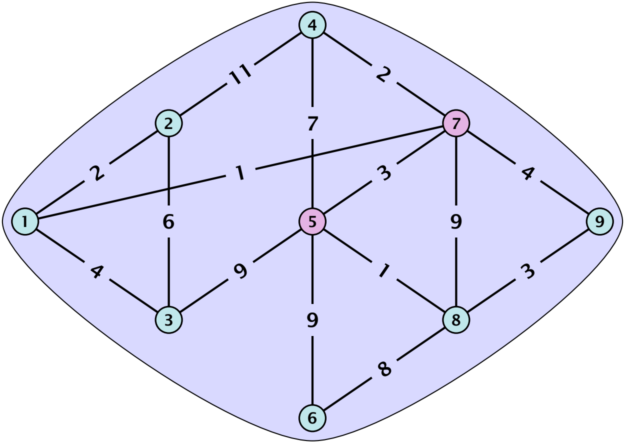

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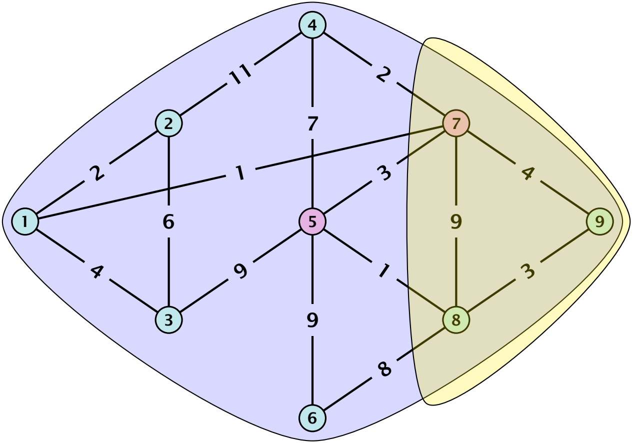

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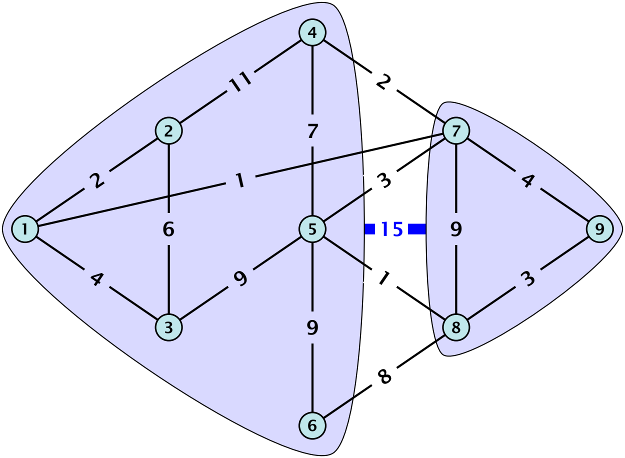

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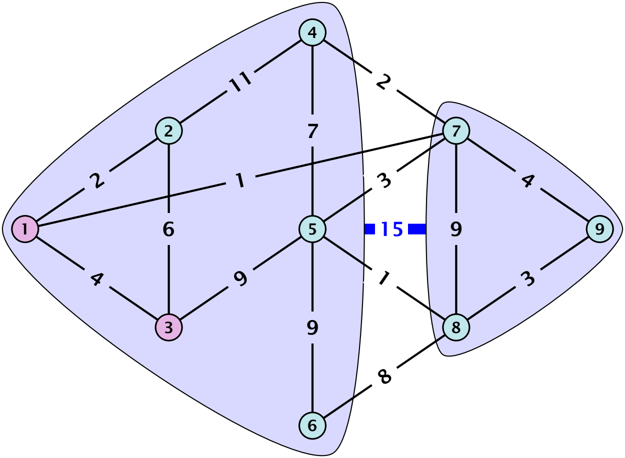

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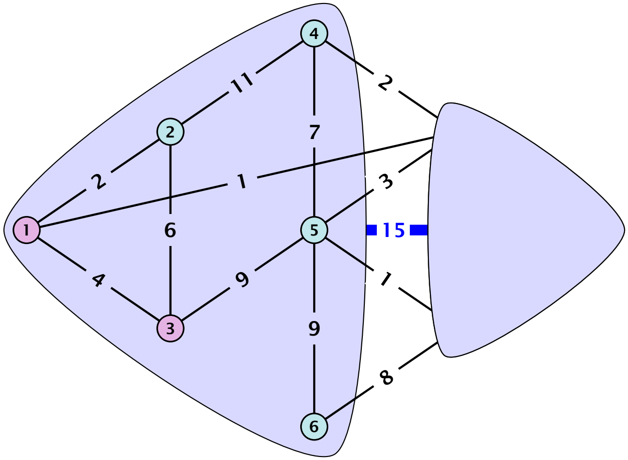

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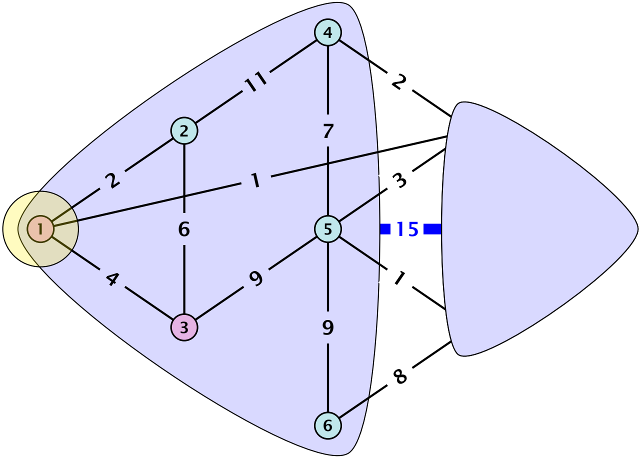

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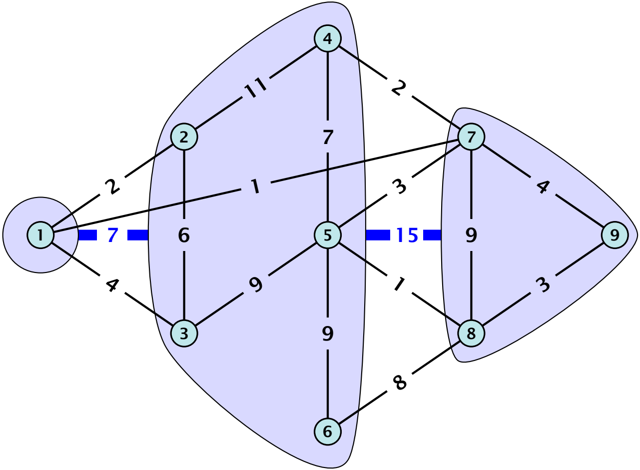

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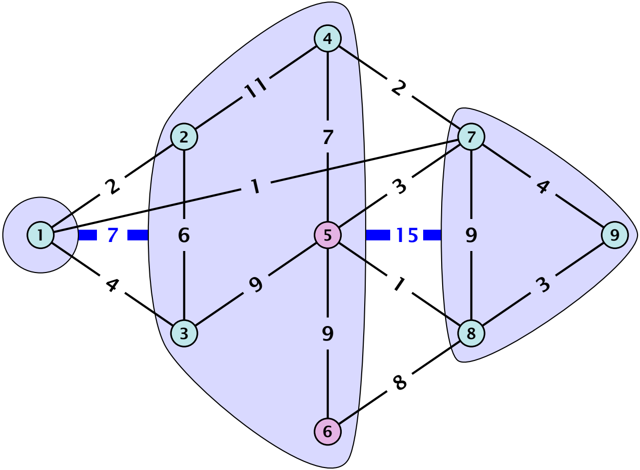

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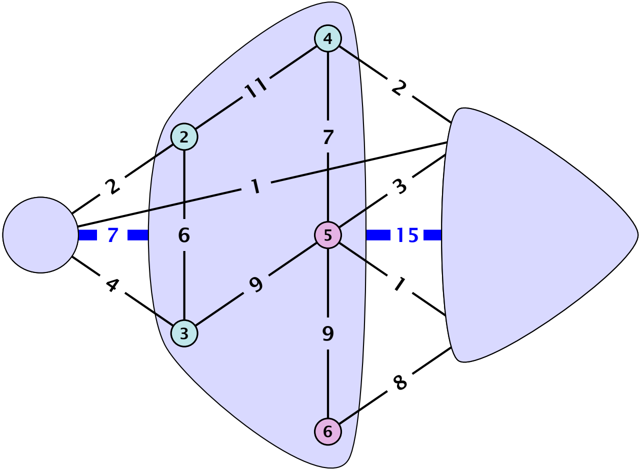

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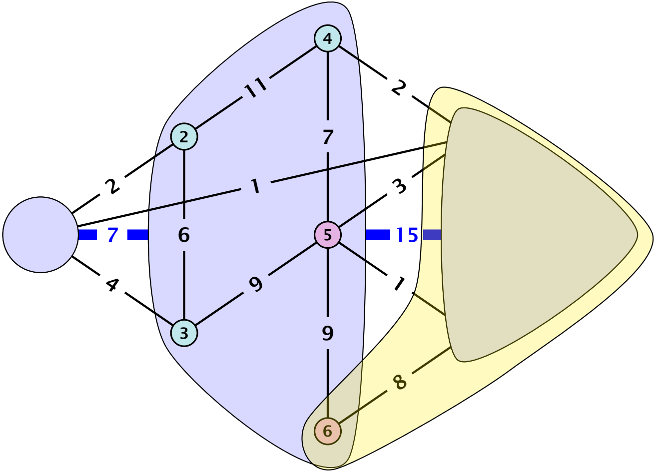

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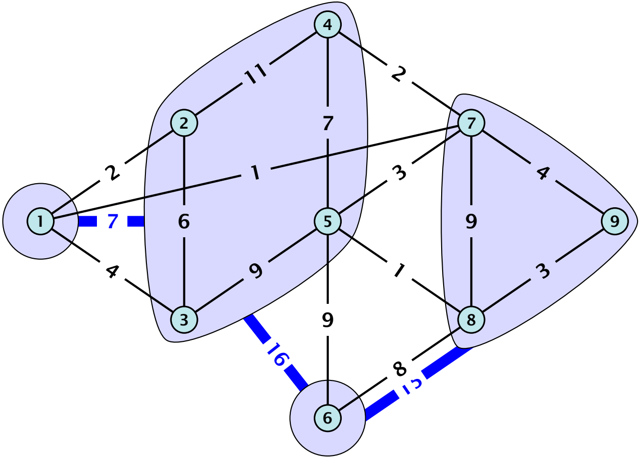

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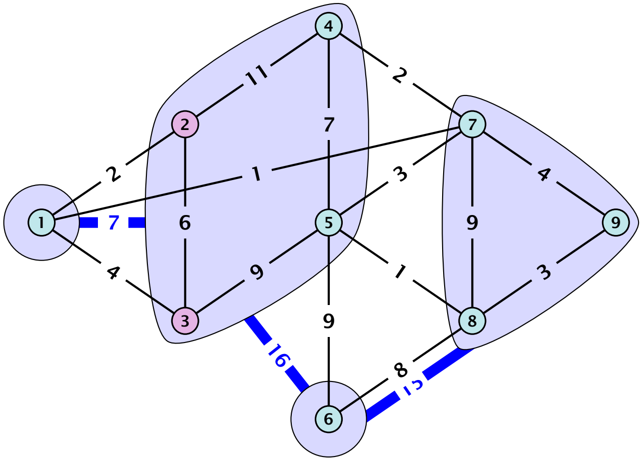

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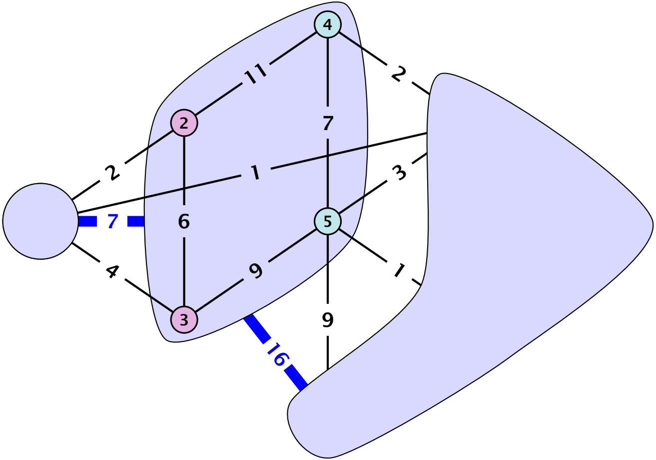

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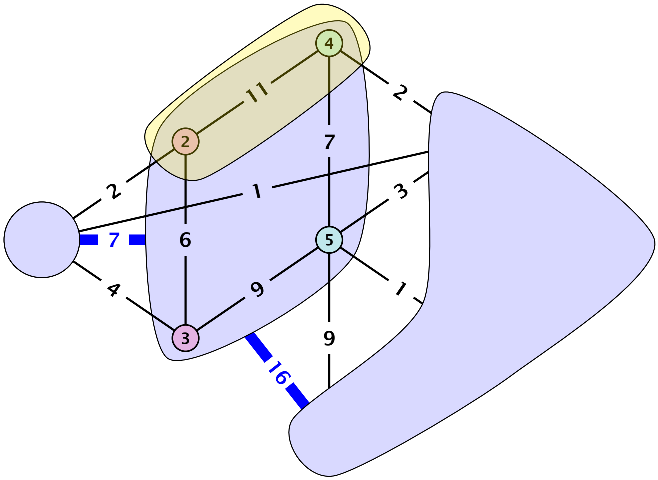

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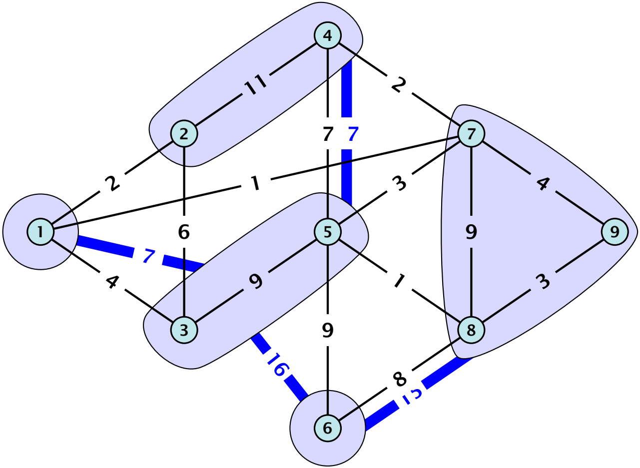

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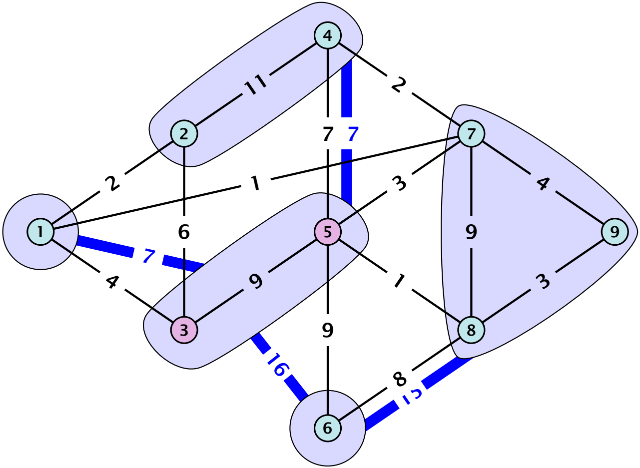

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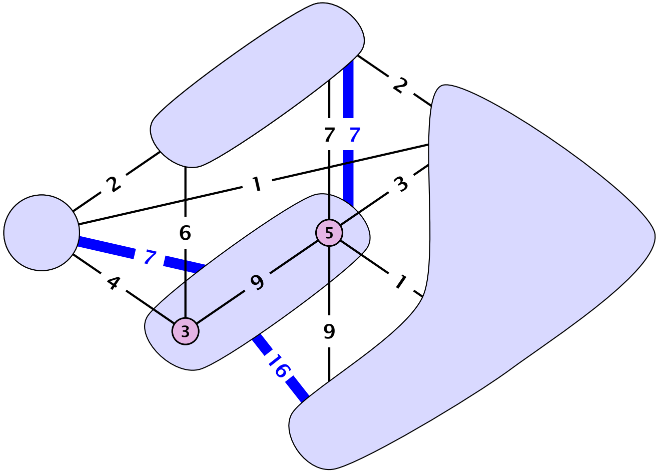

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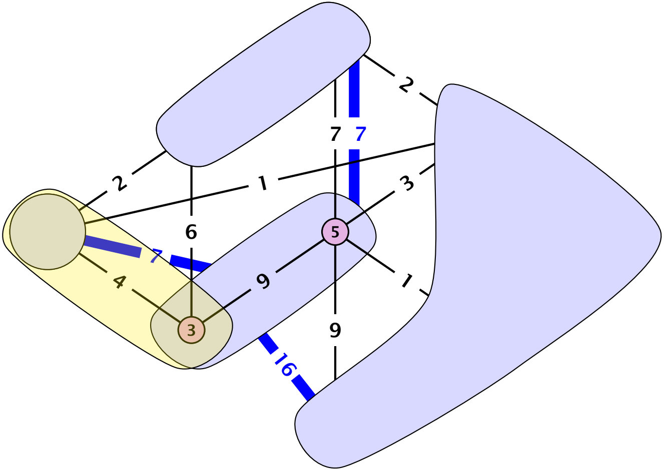

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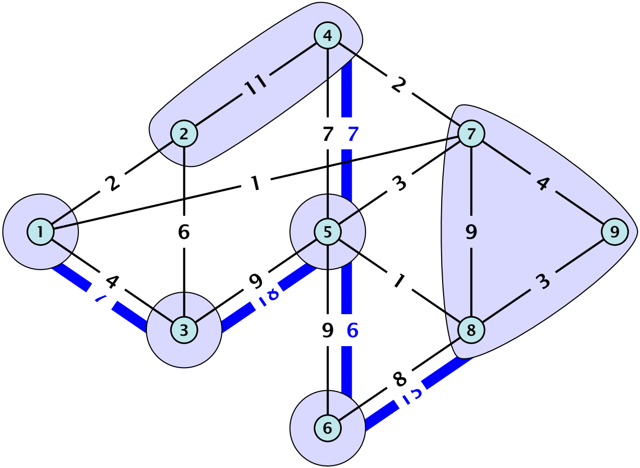

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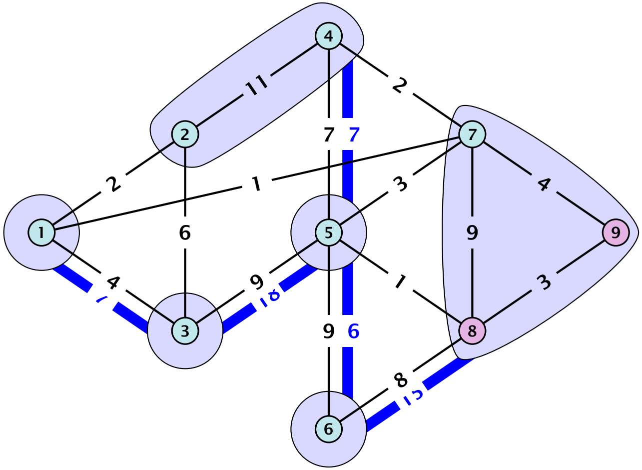

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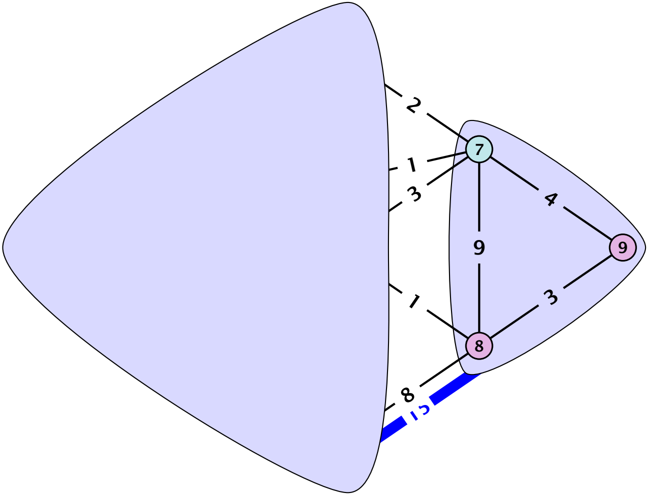

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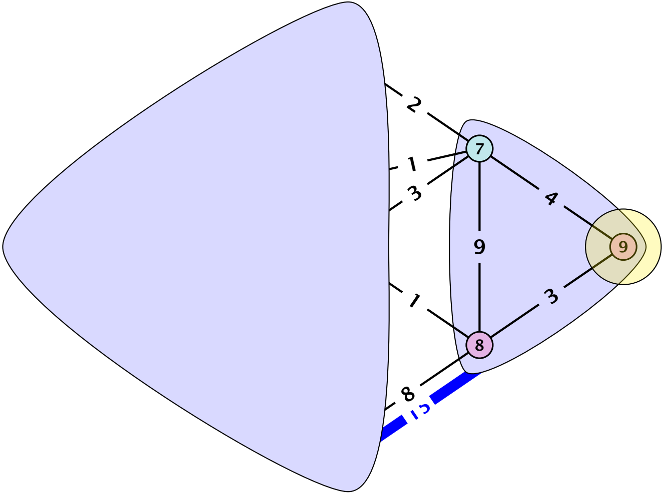

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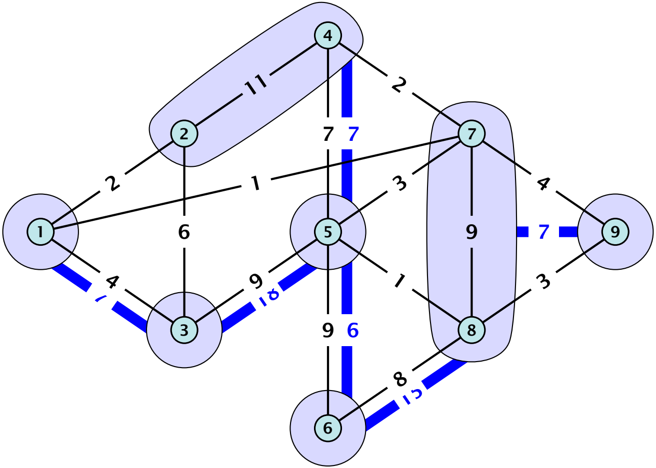

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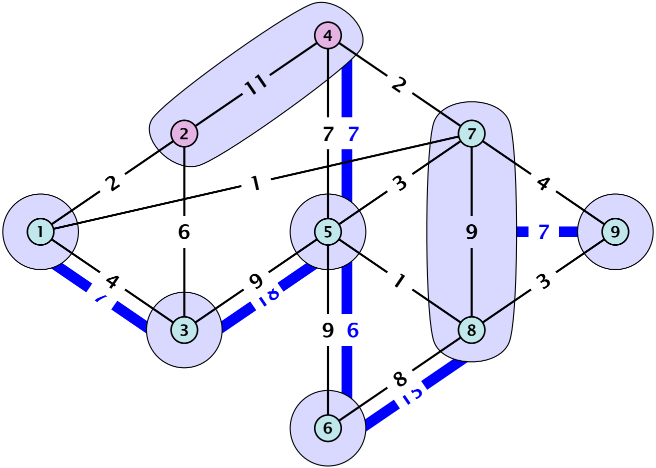

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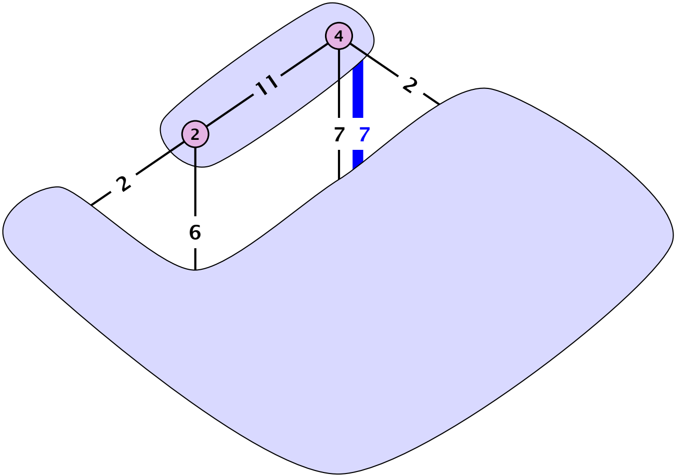

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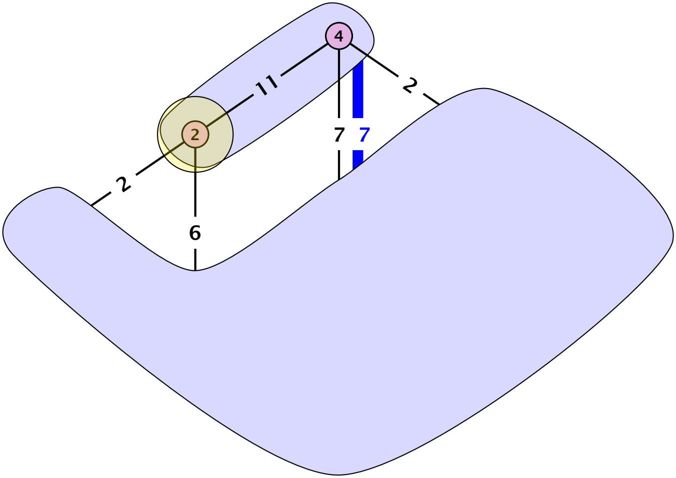

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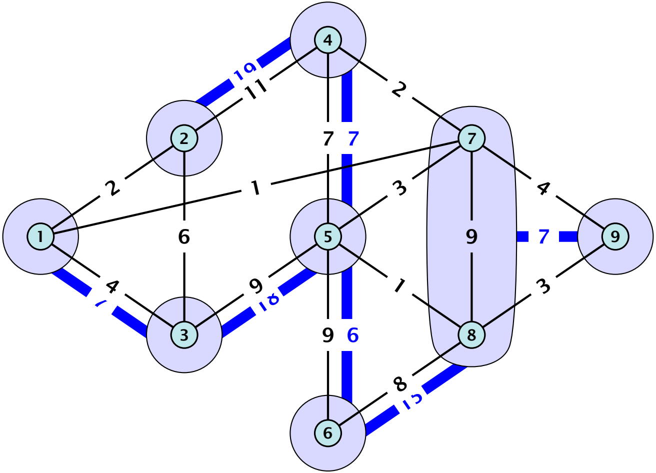

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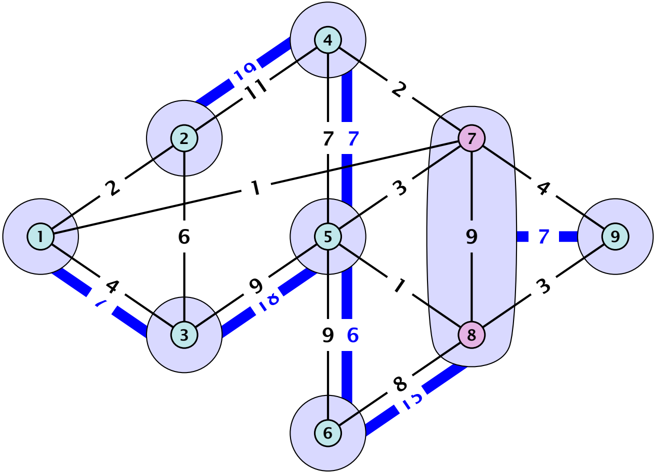

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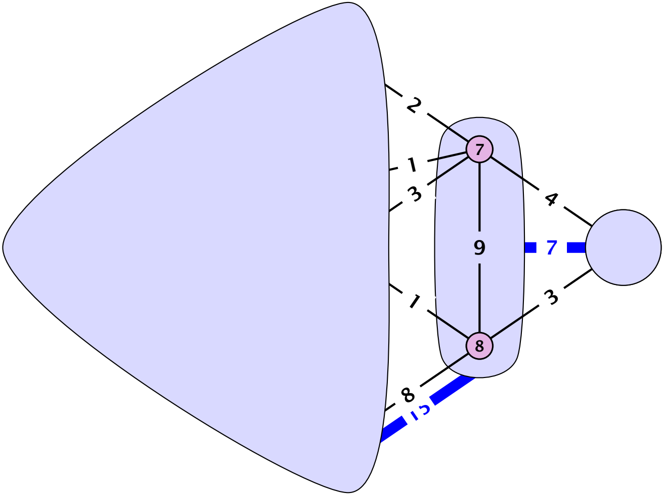

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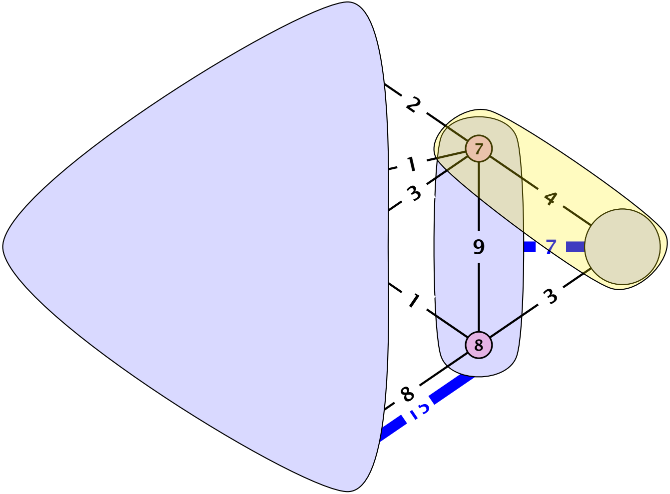

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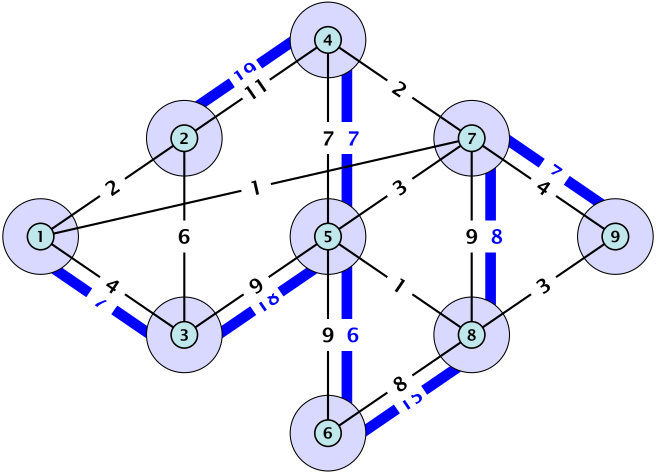

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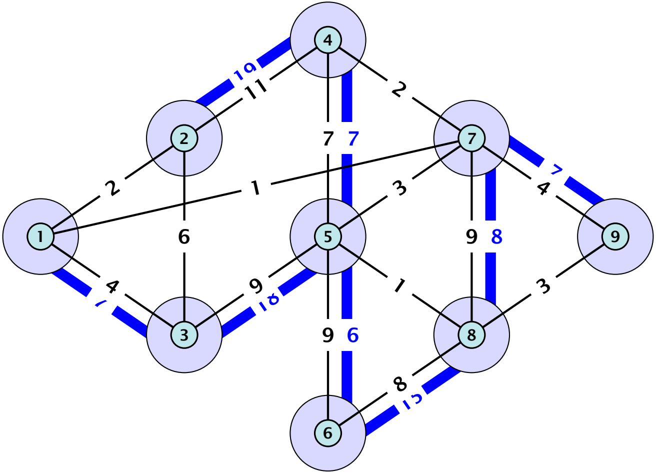

Analysis

Lemma 92

For nodes $s, t, x \in V$ *we have* $f(s,t) \geq \min\{f(s,x), f(x,t)\}$

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Lemma 94

Let S be some minimum r-s cut for some nodes $r, s \in V$ ($s \in S$), and let $v, w \in S$. Then there is a minimum v-w-cut T with $T \subset S$.

Proof: Let X be a minimum v-w cut with $X \cap S \neq \emptyset$ and $X \cap (V \setminus S) \neq \emptyset$. Note that $S \setminus X$ and $S \cap X$ are v-w cuts inside S.

We may assume w.l.o.g. $s \in X$.

First case $r \in X$.

- $\text{cap}(X \setminus S) + \text{cap}(S \setminus X) \leq \text{cap}(S) + \text{cap}(X)$.
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Second case $r \notin X$.

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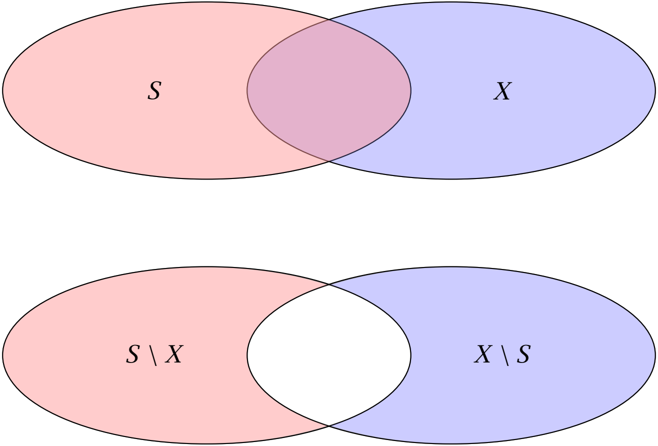

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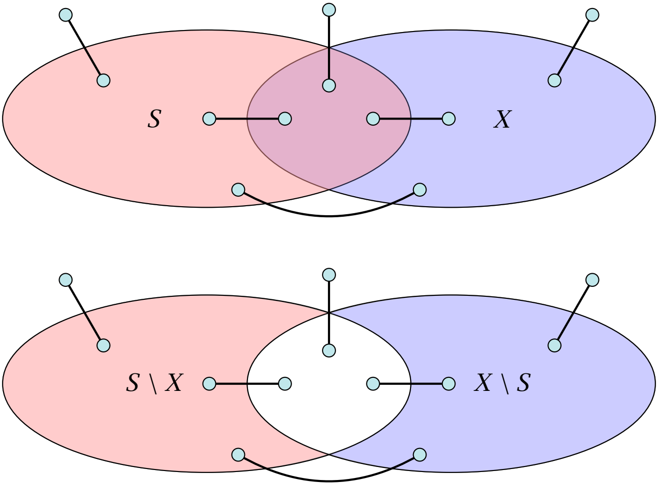

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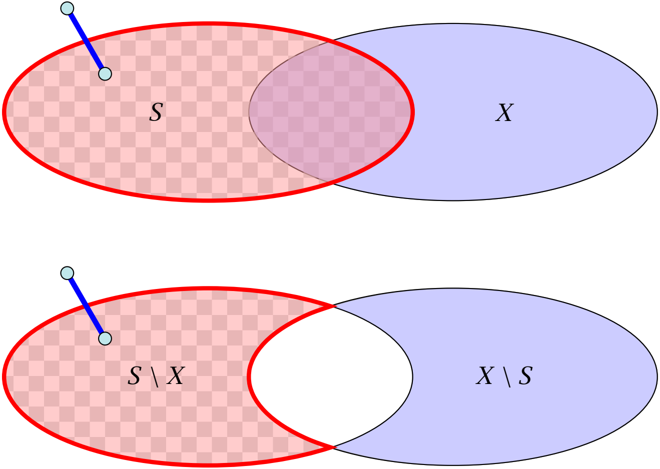

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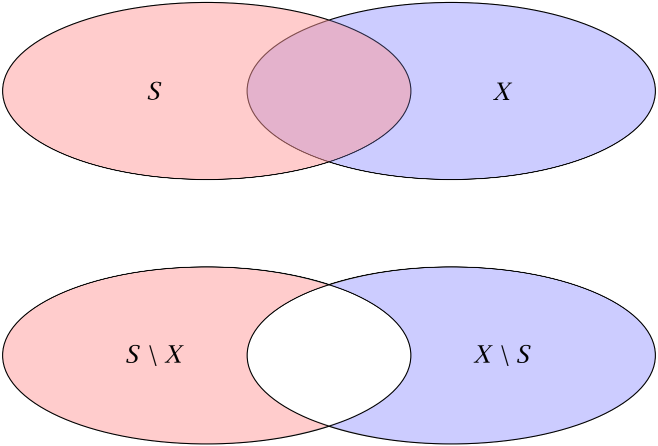

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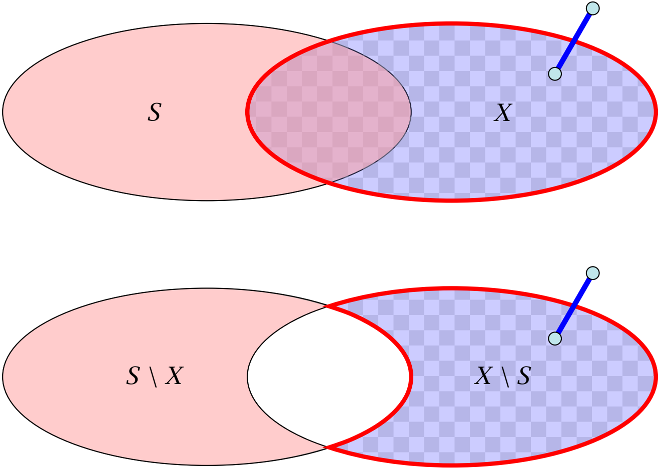

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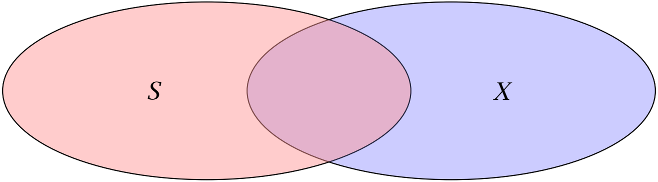

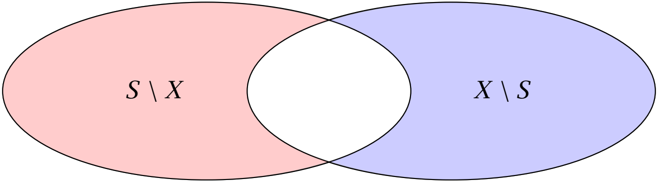

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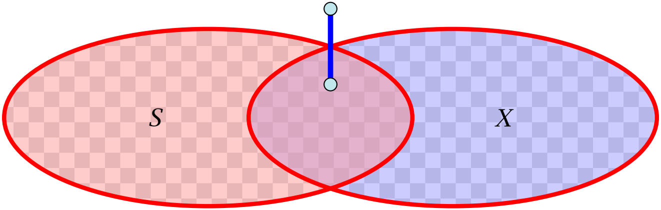

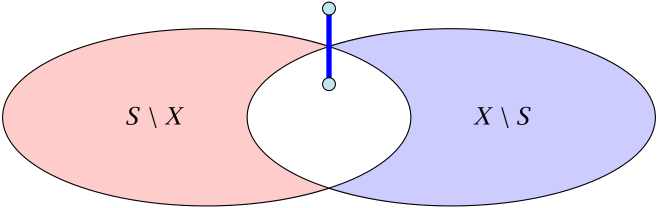

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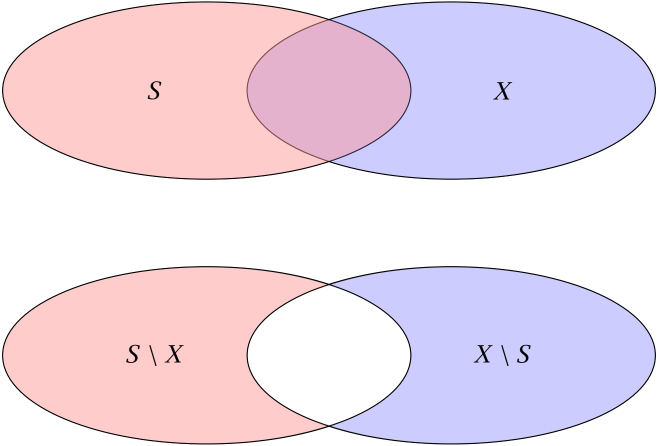

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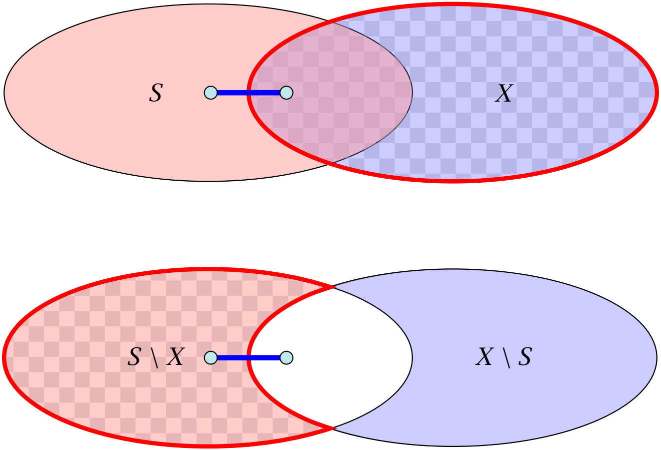

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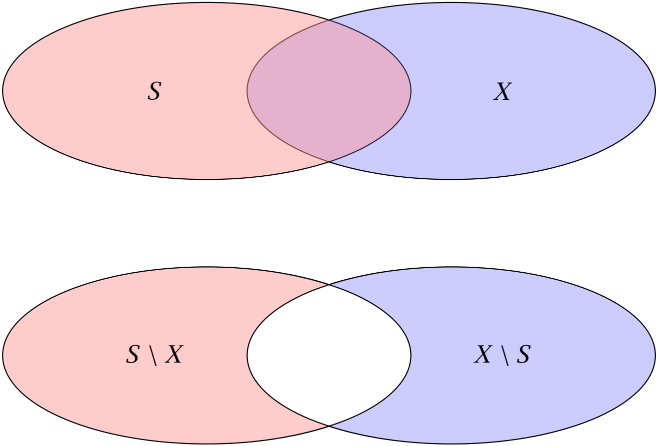

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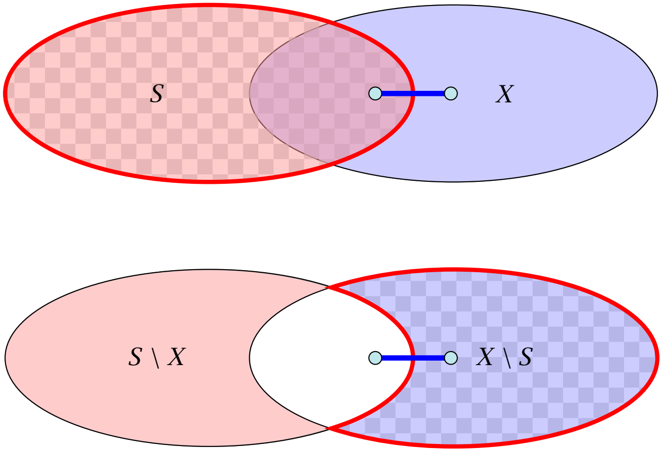

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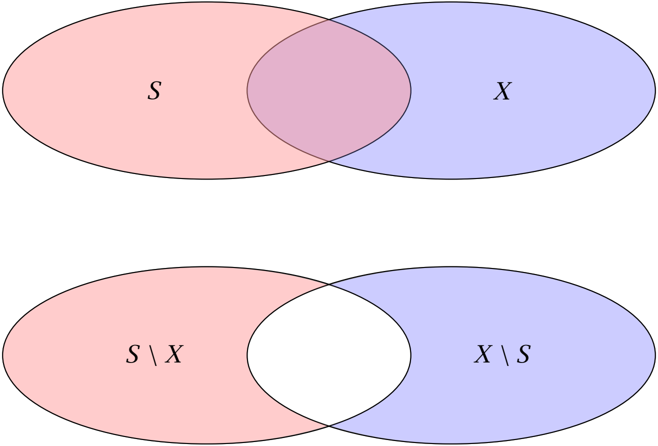

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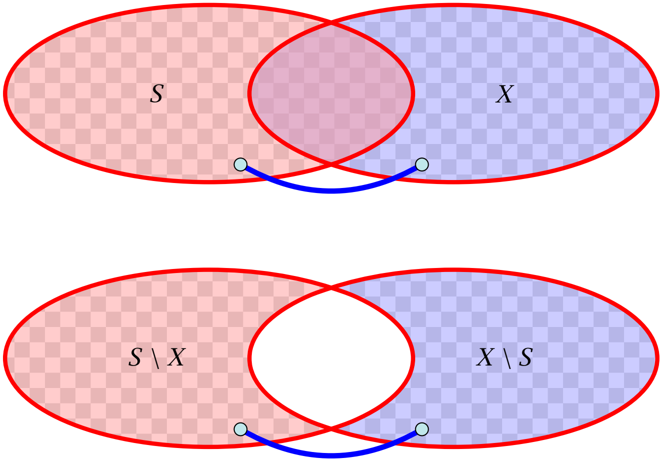

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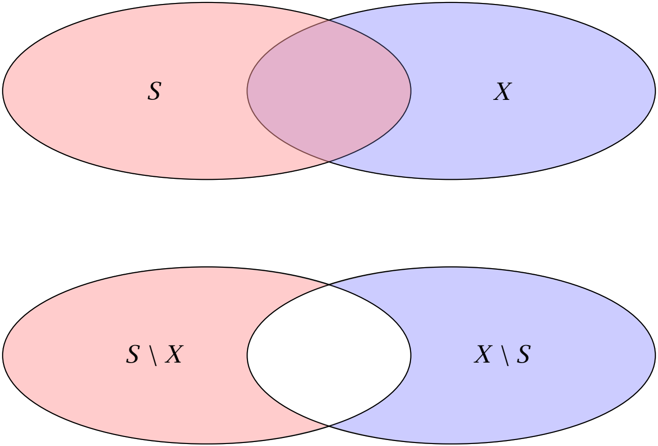

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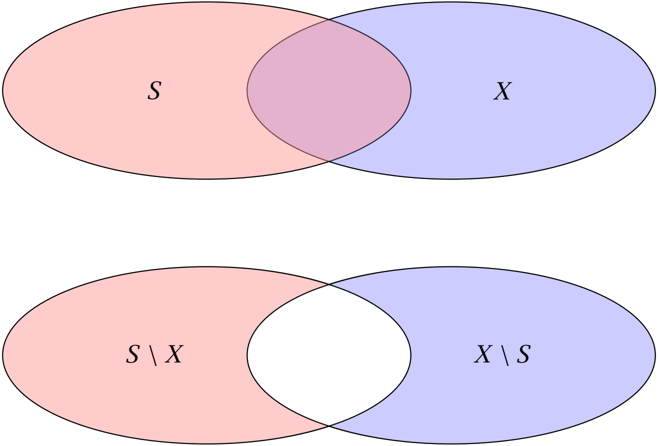

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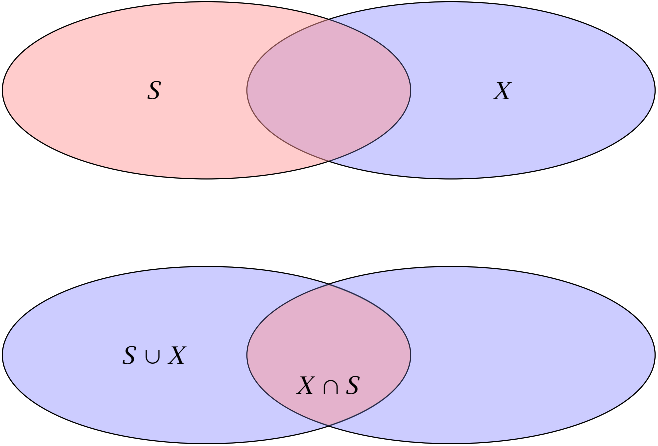

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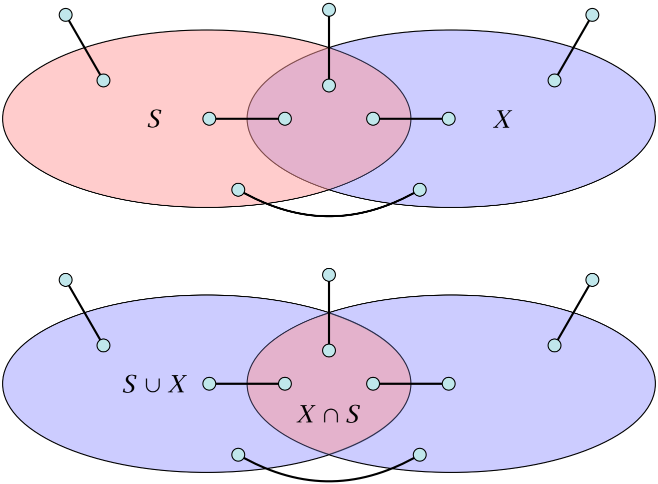

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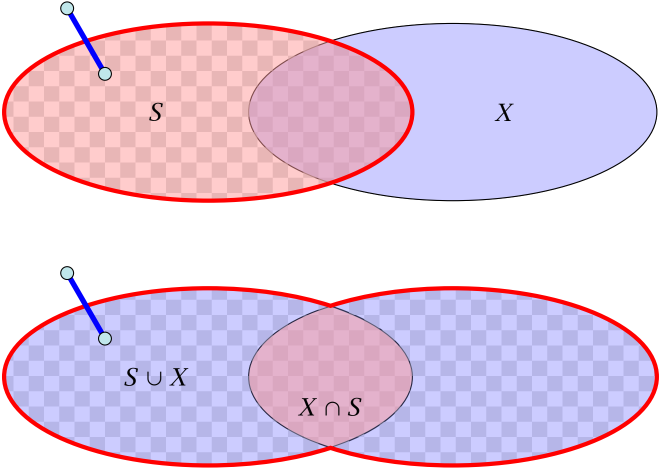

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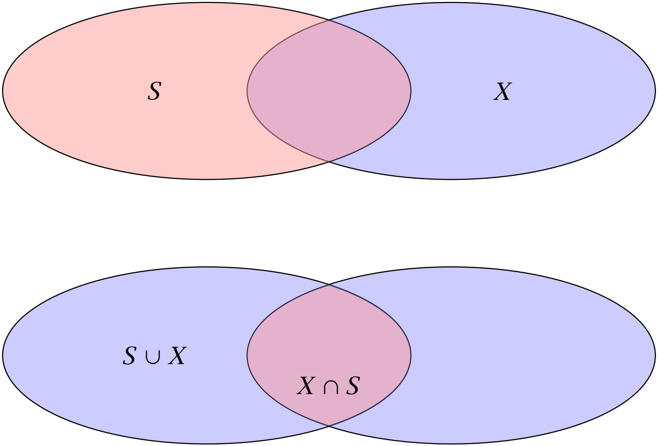

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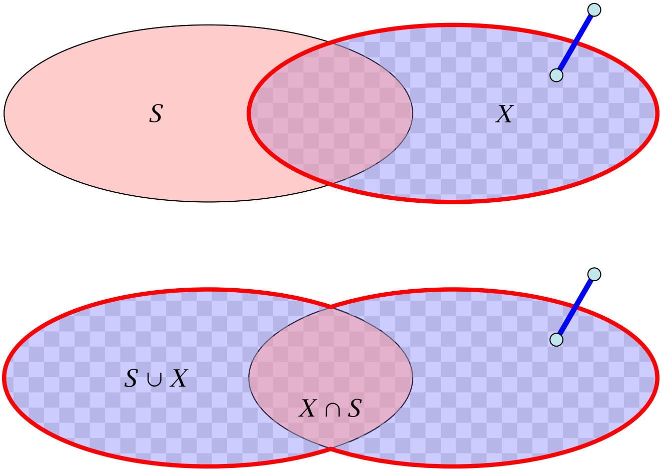

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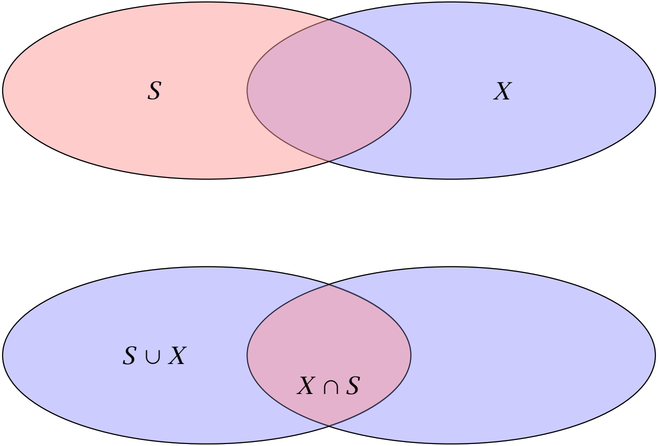

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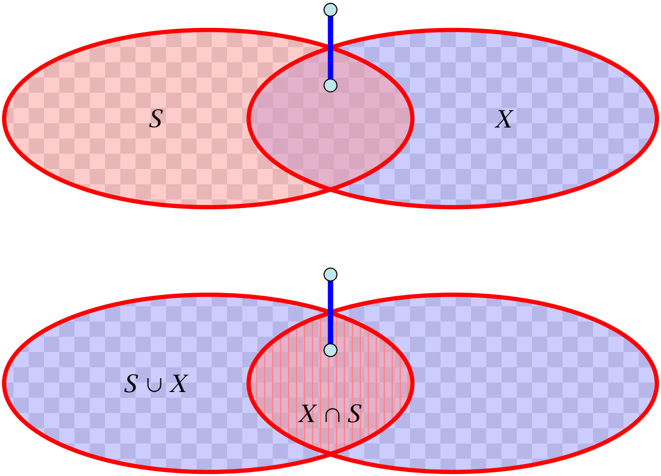

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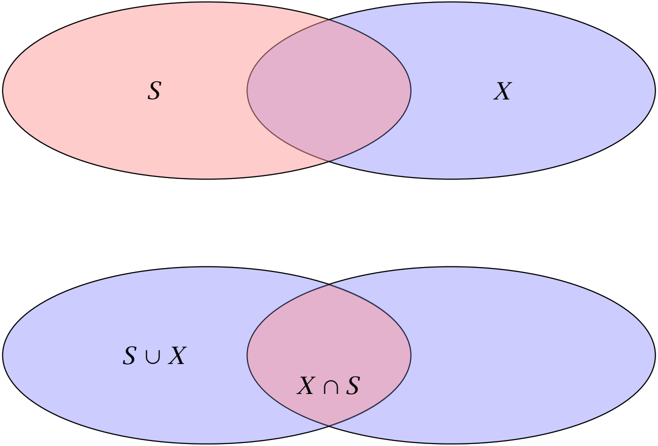

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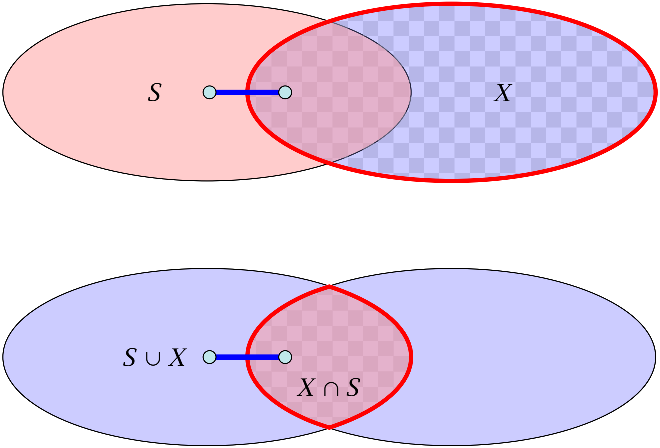

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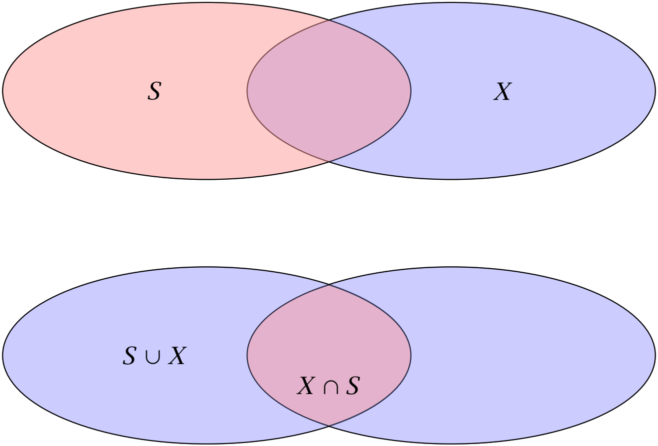

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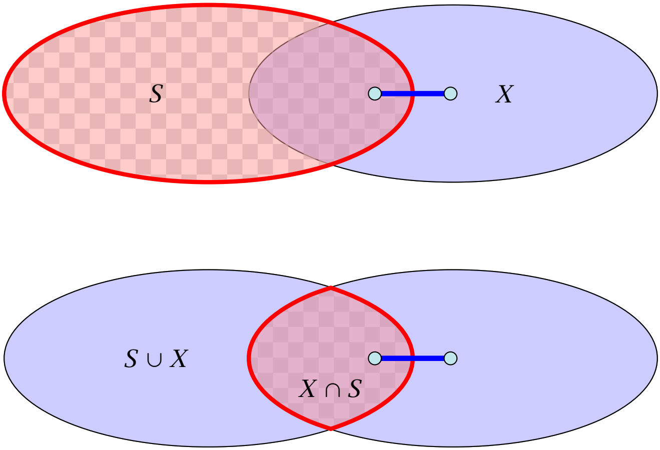

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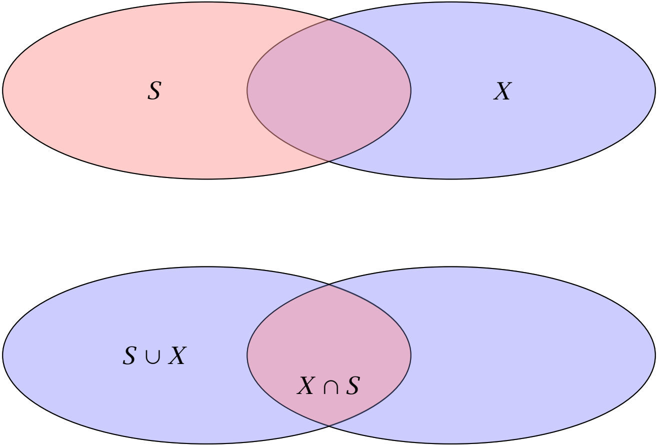

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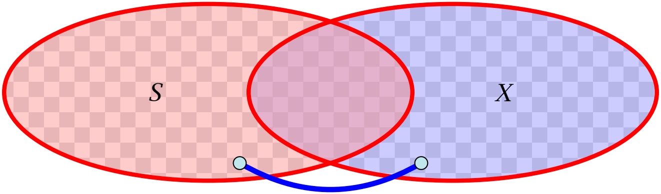

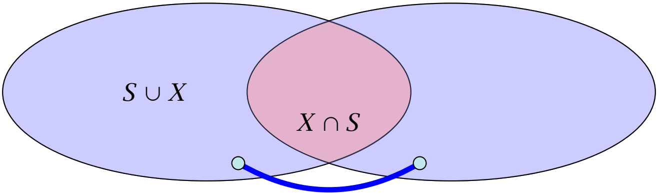

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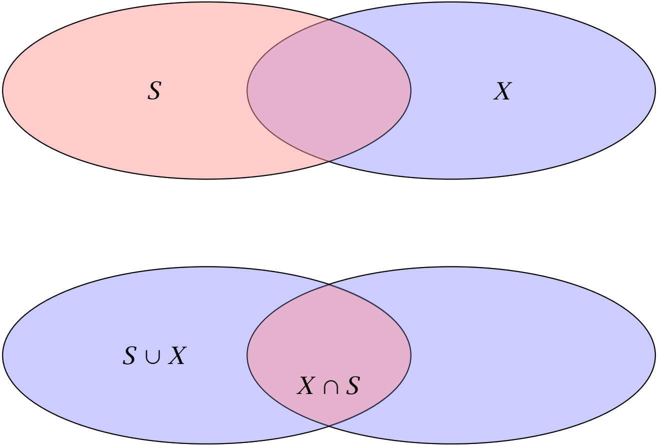

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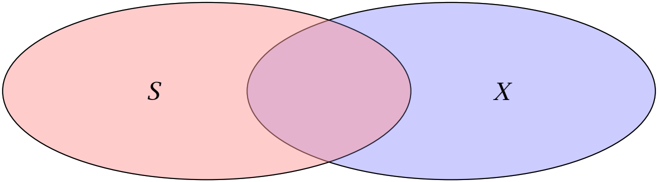

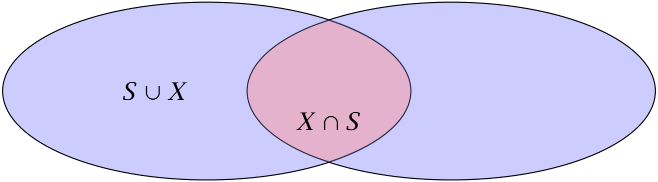

Analysis

Lemma 94 tells us that if we have a graph $G = (V, E)$ and we contract a subset $X \subset V$ that corresponds to some mincut, then the value of $f(s,t)$ does not change for two nodes $s, t \notin X$.

We will show (later) that the connected components that we contract during a split-operation each correspond to some mincut and, hence, $f_H(s,t) = f(s,t)$, where $f_H(s,t)$ is the value of a minimum s-t mincut in graph H.

Analysis

Invariant [existence of representatives]:
For any edge $\{S_i, S_j\}$ in T, there are vertices $a \in S_i$ and $b \in S_j$ such that $w(S_i, S_j) = f(a, b)$ and the cut defined by edge $\{S_i, S_j\}$ is a minimum a-b cut in G.

Analysis

We first show that the invariant implies that at the end of the algorithm T is indeed a cut-tree.

Analysis

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- Since by the invariant this edge induces an s-t cut with capacity $f(x_j, x_{j+1})$ we get $f(s,t) \le f(x_j, x_{j+1}) = f_T(s,t)$.

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- Hence, $f_T(s,t) = f(s,t)$ (flow equivalence).
- The edge $\{x_j, x_{j+1}\}$ is a mincut between s and t in T.
- By invariant, it forms a cut with capacity $f(x_j, x_{j+1})$ in G (which separates s and t).
- Since, we can send a flow of value $f(x_j, x_{j+1})$ btw. s and t, this is an s-t mincut (cut property).

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Proof of Invariant

The invariant obviously holds at the beginning of the algorithm.

Now, we show that it holds after a split-operation provided that it was true before the operation.

Let S_i denote our selected cluster with nodes a and b. Because of the invariant all edges leaving $\{S_i\}$ in T correspond to some mincuts.

Therefore, contracting the connected components does not change the mincut btw. a and b due to Lemma 94.

After the split we have to choose representatives for all edges. For the new edge $\{S_i^a, S_i^b\}$ with capacity $w(S_i^a, S_i^b) = f_H(a,b)$ we can simply choose a and b as representatives.

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The set B forms a mincut separating a from b. Contracting all nodes in this set gives a new graph G' where the set B is represented by node v_B. Because of Lemma 94 we know that $f'(x,a) = f(x,a)$ as $x, a \notin B$.

We further have $f'(x,a) \geq \min\{f'(x,v_B), f'(v_B,a)\}$.

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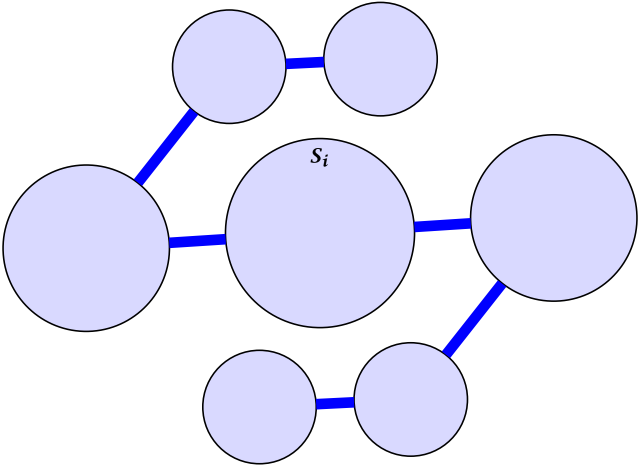

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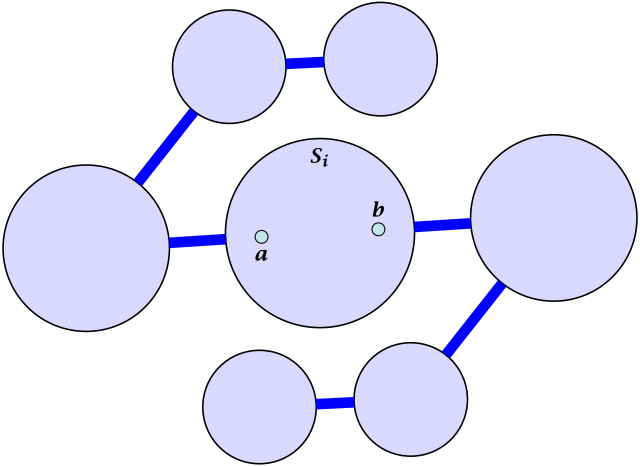

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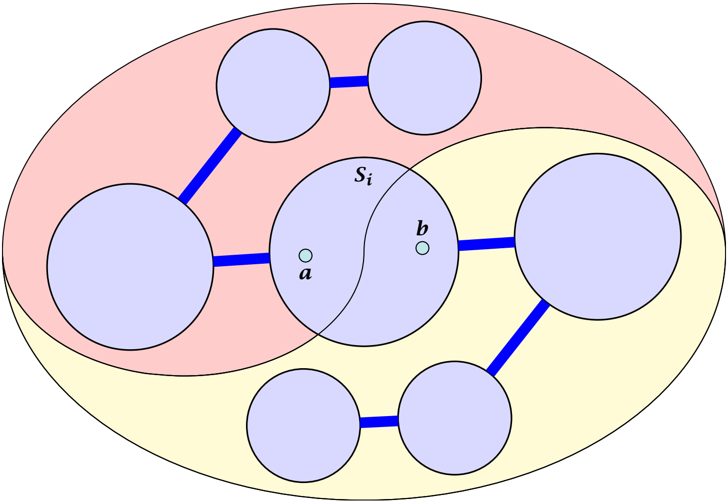

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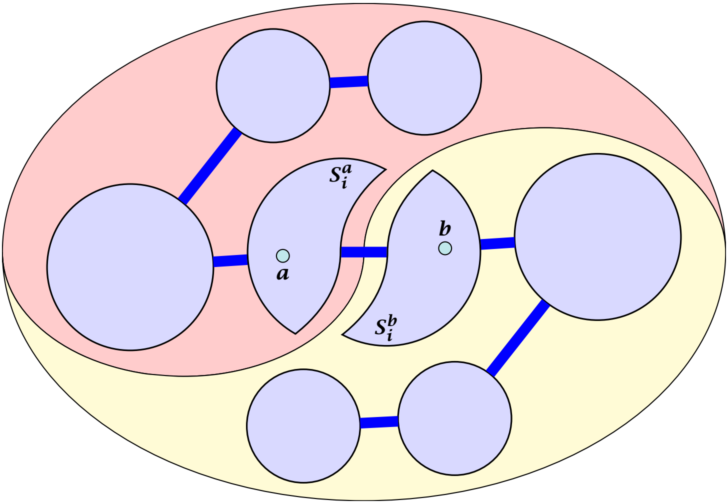

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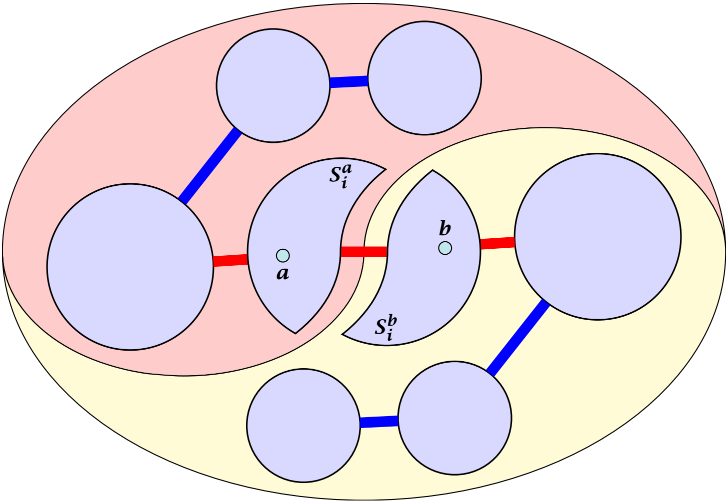

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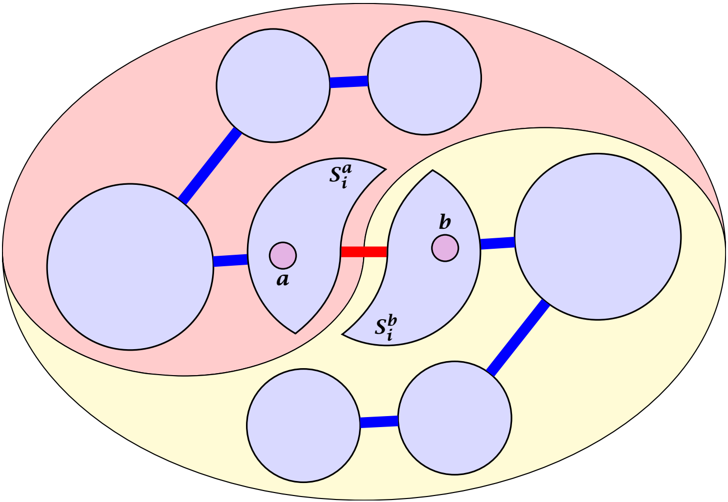

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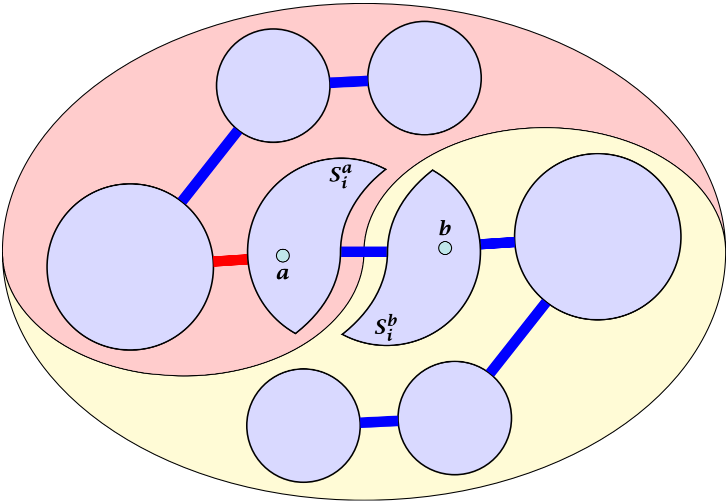

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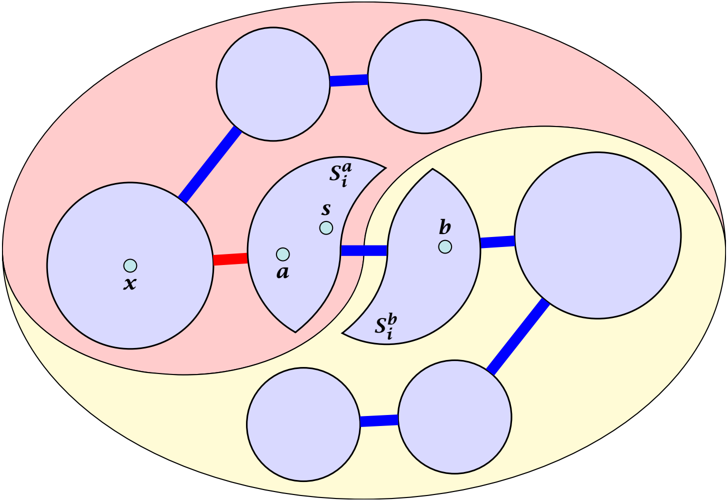

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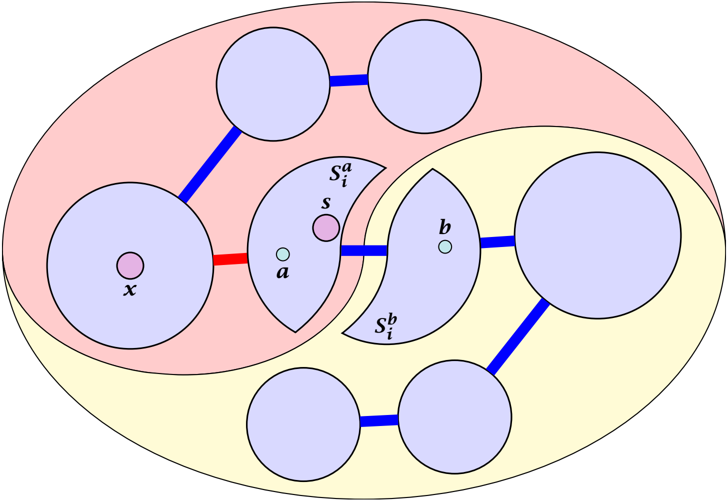

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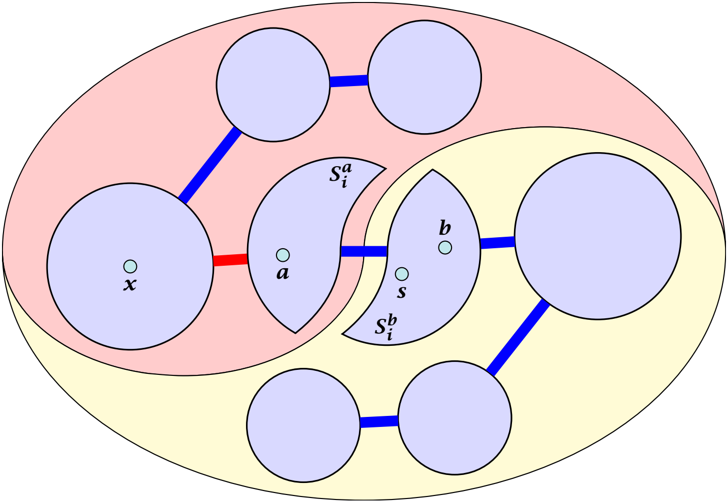

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