Praktikum Diskrete Optimierung

Due date: Monday, 21th May 2012, 12:00

Aufgabe 1 (Maximum Matching in bipartite graphs `hopcroft`)

Let $G = (V, E)$ be an undirected bipartite graph such that $V = V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$ and every edge is adjacent to a node of V_1 and a node of V_2. Implement the algorithm of Hopcroft and Karp which computes a matching of maximal cardinality in time $O(\sqrt{|V|} \cdot |E|)$.

Utilize the visualization capabilities of `GraphWin` to vividly visualize how the algorithm works, that is to vividly visualize each simultan BFS and the respective following DFS. At each point in time the temporary matching should be clearly visible. Also the inverting along augmenting paths should be visualized appropriately.

Additionally, before each simultan BFS the upper bound $2 \cdot \left\lfloor \frac{|M|}{|M'|-|M|} \right\rfloor + 1$ of the length of a shortest augmenting path should be printed out, where M is the current matching and M' is some maximum matching. Since M' is unknown, the upperbound $|V|/2$ should be used for $|M'|$.

Hints

You can use the graphs `bipartite1.gw` to `bipartite4.gw` as inputs for your algorithm. In this graphs all nodes of V_1 contain the user label "1" while the nodes of V_2 contain the user label "2". Each of these graphs contain a perfect matching, i.e. a matching having $|V|/2$ edges.