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Problem Set 3
April 30, 2013

Complexity Theory

Due date: May 7, 2013 **before** *class!*

Problem 1 (10 Points)

Consider a graph $G = (V, E)$. Recall the following definitions from the lecture:

- A *clique* is defined as a subset $V' \subseteq V$ of vertices such that the induced subgraph of V' is complete, i.e. all vertices in V' are pairwise connected with edges.
  Let $\textsc{Clique} = \{(G, k) : \text{the graph } G \text{ has a clique of } k \text{ vertices}\}$.
- An *independent set* is defined as a subset $V' \subseteq V$ of vertices such that no two vertices of V' are connected by an edge.
  Let $\textsc{Indset} = \{(G, k) : \text{the graph } G \text{ has an independent set of } k \text{ vertices}\}$.

Show the following:

(i) $\textsc{Indset} \preceq_m^p \textsc{Clique}$,

(ii) $\textsc{Clique} \preceq_m^p \textsc{Indset}$,

(iii) $3\textsc{Sat} \preceq_m^p \textsc{Clique}$,

(iv) $\textsc{Clique}$ is $\mathcal{NP}$-complete.

Problem 2 (10 Points)

Consider the problem of *map coloring*: Can you color a map with k different colors, such that no pair of adjacent countries has the same color?

(i) Describe the map coloring problem as a proper graph problem and redefine the language k-$\textsc{Colorability} = \{\text{Maps that are colorable with at most } k \text{ colors}\}$.

(ii) Show that 2-$\textsc{Colorability}$ is in $\mathcal{P}$.

(iii) Show that 3-$\textsc{Colorability}$ is $\mathcal{NP}$-complete.
Hint: Use a reduction from $3\textsc{sat}$.

Problem 3 (10 Points)

Recall the following definition: A language A is *polynomial-time Cook-reducible* to a language B if there is a polynomial-time TM M that, given an oracle deciding B, can decide A. (An oracle for B is a TM that can in decide membership in B in $\mathcal{O}(1)$ time.) Show that $3\textsc{Sat}$ is Cook-reducible to $\textsc{Tautology}$.

Problem 4 (10 Points)

In the Exactly One 3Sat problem, we are given a 3CNF formula φ and need to decide if there exists a satisfying assignment u for φ such that every clause of φ has exactly one TRUE literal. Prove that Exactly One 3Sat is $\mathcal{NP}$-complete.