Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
Prof. Dr. Ernst W. Mayr
Chris Pinkau

Spring Semester 2013
Problem Set 6
June 4, 2013

Complexity Theory

Due date: June 11, 2013 **before** *class!*

Problem 1 (10 Points)

Define the class **DP** $= \{L = L_1 \cap L_2 : L_1 \in \mathcal{NP}, L_2 \in \text{co}\mathcal{NP}\}$. (Note that we do not know if **DP** $= \mathcal{NP} \cap \text{co}\mathcal{NP}$.) Consider the following languages:

$$\textsc{ExactIndset} = \{(G, k) : \text{ the largest independent set of } G \text{ has size exactly } k\},$$
$$\textsc{Critical Sat} = \{\varphi : \varphi \text{ is unsatisfiable, but deleting any clause makes it satisfiable}\}.$$

Show the following:

(i) $\textsc{ExactIndset} \in \Sigma_2^p = \mathcal{NP}^{\mathcal{NP}}$.

(ii) $\textsc{ExactIndset} \in$ **DP**.

(iii) $\textsc{Critical Sat}$ is **DP**-complete.

Problem 2 (10 Points)

Recall the definition of alternating Turing machines (ATM) with control states partitioned into sets $Q_\forall$ and $Q_\exists$, and the corresponding class **AP**.

(i) Show that a language $L \in$ **AP** decided by an *existential* ATM (i.e. $Q_\forall = \emptyset$) is in $\mathcal{NP}$.

(ii) Show that a language $L \in$ **AP** decided by an *universal* ATM (i.e. $Q_\exists = \emptyset$) is in co$\mathcal{NP}$.

(iii) Show that **AP** = co-**AP**.

(iv) Show that **PSPACE** is contained in **AP** by showing that $\textsc{tQbf} \in$ **AP**.

Problem 3 (10 Points)

Prove **AL** $= \mathcal{P}$.

Problem 4 (10 Points)

(i) Argue that at least one of the assumptions $\mathbf{L} \neq \mathcal{P}$ and $\mathcal{P} \neq$ **PSPACE** is true.

(ii) Use padding to show that if $\mathcal{P} = \mathbf{L}$, then **EXP** = **PSPACE**.