Technische Universität München
Fakultät für Informatik
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Problem Set 11
January 28th, 2014

Parallel Algorithms

Due date: February 4th, 2014 **before** *class!*

Problem 1 (10 Points)

Consider the complete bipartite graph $K_{2,3}$ and its shortest path metric d. Show that this metric are not ℓ_1-embeddable. Exploit the correspondence between ℓ_1-metrics and cuts. Show that distributions over cuts cannot generate the correct distances by differentiating between pairs that are at distance 1 (short distances) and at distance 2 (long distances), respectively.

Problem 2 (20 Points)

An *expander graph* $G = (V, E)$ has the property that the number of edges in any cut is some constant factor times the smaller side of the cut; i.e., there is some constant $\alpha > 0$ such that for any $S \subseteq V$ such that $|S| \leq |V|/2$, $|\delta(S)| \geq \alpha|S|$. There exist expanders such that every vertex in G has degree three.

1. Given an expander $G = (V, E)$ with $c_e = 1$ for all $e \in E$, an (s_i, t_i) pair for each $j, k \in V$ with $j \neq k$ and $D_i = 1$ for all i, show that the sparsest cut has value at least $\Omega(1/n)$.

2. Let G be an expander such that every vertex has degree three. Let (V, d) be the shortest path distance in G; i.e., d_{uv} is the shortest path from u to v in G in which every edge has length 1. Show that for any vertex v, there are at most $n/4$ vertices u such that $d_{uv} \leq \log n - 3$.

3. Given the graph and distance metric from the previous part, show that
$$\frac{\sum_{(u,v)\in E} d_{uv}}{\sum_{u,v \in V: u \neq v} d_{uv}} = \mathcal{O}\left(\frac{1}{n \log n}\right).$$

4. Using the parts above, show that any expander with every vertex having degree three cannot be embedded into ℓ_1 with distortion less than $\Omega(\log n)$.