Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
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Abschlussklausur
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Automaten und formale Sprachen

Last Name	First Name	Matriculation No.	Signature
....	....	....	....

General Information

- Please fill in the fields above and write your name and matriculation number on all extra supplementaries provided.
- Please keep your student's card and an identity card available.
- Do not use pencils! Do not use red or green ink!
- You are not allowed to use any device other than your pens and a double-sided handwritten A4 sized paper.
- You have 180 minutes to answer the questions.

Left Lecture Hall from ...... to ...... / from ...... to ......

Submitted early at ......

Special notes:

	A1	A2	A3	A4	A5	A6	A7	A8	Σ	Examiner
Points	8	5	6	6	10	6	10	9	60	
1^{st} correction										
2^{nd} correction										

Problem 1 (8 Points)

Answer the following questions in one or two short sentences. If the answer is 'yes' or 'no' please justify your choice briefly.

a) Which words does the language $\mathcal{L}(\emptyset^*)$ contain?

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b) Which words does the ω-language $\mathcal{L}(\emptyset^\omega)$ contain?

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c) Why do regular languages have finitely many residuals?

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d) Are finite ω-languages always ω-regular?

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e) Are finite languages of finite words always regular?

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f) Are regular languages equivalent to type-0 languages in the Chomsky hierarchy?

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g) Are DBAs and NRAs equally expressive?

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h) Are the following statements equivalent?
(1) The set of states $F = \{q_1, q_2, ..., q_k\}$ is visited infinitely often
(2) $\exists i \in \{1, ..., k\} : q_i$ is visited infinitely often

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i) **Bonus-question:** In which year was Mojzesz Presburger born and who mentored his MA-Thesis?

Problem 2 (5 Points)

Prove or disprove:

(a) $L_1 = \{w \in \{a, b\}^* \mid abw = wba\}$ is regular.

(b) $L_2 = \{a^n b^m \mid n \leq 10^9 \wedge m \leq 10^n\}$ is regular.

(c) $L_3 = \{w \in \{a, b, (,)\}^* \mid$ The numbers of opening and closing brackets in w are equal$\}$ is regular.

(d) $L_4 = \{w \in \{a, b, c\}^\omega \mid$ If $a \in \inf(w)$ then $c \notin \inf(w)\}$ is ω-regular.

(e) $L_5 = \{w \in \{a, b, c\}^\omega \mid$ For all finite prefixes v of w the number of as in v equals the number of bs in v.$\}$ is ω-regular.

Remarks:

- A finite automaton recognizing a given language is regarded as a proof for regularity.
- You may use the fact that $\{a^n b^n \mid n \in \mathbb{N}\}$ is not regular.

Problem 3 (6 Points)

Consider the following regular expressions:

- $r_1 = ab^*(a+b)^*c$
- $r_2 = a(a+bc+c^*)^*a$
- $r_3 = \Sigma^*(abc+bca+cab)\Sigma^*$

(a) Describe in words the language induced by each regular expression above.

(b) Construct a finite automaton (NFA or DFA) for each regular expression above.

(c) Give an MSO-sentence for each regular expression above.

Problem 4 (6 Points)

The derivative of a language $L \subseteq \Sigma^*$ with respect to a symbol $a \in \Sigma$ is defined as:

$$\frac{\delta L}{\delta a} = \{w \mid aw \in L\}$$

(a) Show that if L is regular then $\frac{\delta L}{\delta a}$ is regular as well.

(b) Let $L_1 \subseteq \Sigma^*$ and $L_2 \subseteq \Sigma^*$ be regular languages. Show how to express the derivative of $L_1 L_2$ with respect to a using the rule for the derivative of a single language above.

Problem 5 (10 Points)

For any given language $L \subseteq \Sigma^*$, let L_{pre} (resp. L_{suf}) denote the language containing all prefixes (resp. all suffixes) of the words in L.

(a) Given a finite automaton A s.t. $\mathcal{L}(A) = L$, construct a finite automaton B s.t. $\mathcal{L}(B) = (L_{pre})_{suf}$.

(b) Let $r = (ab+a)^*c$ be a regular expression over $\Sigma = \{a, b, c\}$. Give a regular expression $r_{pre,suf}$ s.t. $\mathcal{L}(r_{pre,suf}) = (\mathcal{L}(r)_{pre})_{suf}$.

Problem 6 (6 Points)

Let $L \subseteq \Sigma^*$ be a regular language. Show how to construct a transducer that accepts

$$L' = \{(a_1 a_2 ... a_n, b_1 b_2 ... b_n) \mid \begin{array}{l} a_1 a_2 ... a_n \in L \quad \wedge \\ b_1 b_2 ... b_n \in L \quad \wedge \\ \exists c \in \Sigma^n : a_1 b_1 c_1 a_2 b_2 c_2 ... a_n b_n c_n \in L\}. \end{array}$$

Explain your construction.

Problem 7 (10 Points)

(a) Let $\Sigma = \{a, b, c\}$. Give an NBA, an NMA and an NRA for each of the following languages:

(1) $L_1 = \{w \mid \text{every } b \text{ and } c \text{ is preceded (not necessarily immediately) by an } a\}$

(2) $L_2 = \{w \mid \text{every } b \text{ is preceded by an } a \text{ and succeeded by a } c\}$
(as before, preceded/succeeded does not necessarily imply immediacy)

(b) Let A and B be two NBAs. We define the shuffle-product of two ω-languages as

$$s(L_1, L_2) = \{w_0^1 w_0^2 w_1^1 w_1^2 ... \mid w^1 \in L_1 \wedge w^2 \in L_2\}.$$

Show how to construct an NBA that recognizes the language $s(\mathcal{L}(A), \mathcal{L}(B))$. Explain your solution.

Problem 8 (9 Points)

Use the method from class to generate a finite automaton recognizing the solution space of the following Presburger formula. Include all intermediate steps. You may merge trap states at any point during the procedure.

$$\forall x : x > 1 \land x + y > 2$$