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Fakultät für Informatik
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Winter Semester 2014/15
Problem set 1
October 07, 2014

Automata and Formal Languages

Due October 14, 2014 before class!

The purpose of this problem set is to repeat basic concepts already acquired in introductory courses.

Exercise 1 (Regular expressions - 10 points)

(a) Describe the regular languages induced by the following regular expressions in your own words:

- $r_1 = 0^*10^*(0^*10^*10^*)$
- $r_2 = 00(0+1)^*$

(b) Decide whether the following languages are regular or not. Justify your decision.

- $\mathcal{L}_1 = \{0^n 1^n \mid n \in \mathbb{N}\}$
- $\mathcal{L}_2 = \{\omega \in \{0,1\}^* \mid \omega \text{ contains the same number of 1s and 0s}\}$

Exercise 2 (Basic Automata - 10 points)

Give an automaton (NFA or DFA) that accepts

(a) All binary strings of length divisible by 5.

(b) All binary strings of length divisible by 3 or 5.
How would you have to change your automaton in order to accept all binary strings of length divisible by 3 **and** 5?

(c) Decimal numbers divisible by 3 or 9.

Exercise 3 (NFA to DFA - 10 points)

Let r be the regular expression $(a(b+c)^*(a+b))$ over the alphabet $\Sigma = \{a,b,c\}$.

(a) Give a non-deterministic finite automaton (NFA) recognizing $\mathcal{L}(r)$.

(b) Convert the NFA from (a) into a DFA using the Myhill-construction.

Exercise 4 (10 points)

(a) Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA with $|Q| = n$. Prove or disprove: If there exists a word ω with $|\omega| > n$ s.t. $\omega \in \mathcal{L}(M)$ then $\mathcal{L}(M)$ is an infinite language.

(b) Let L_1 and L_2 denote regular languages. We define the *zipper-product* (sometimes also referred to as *shuffle-product*) as

$$L_1 \% L_2 = \{a_1 b_1 a_2 b_2 ... a_n b_n \ \mid a_1 a_2 ... a_n \in L_1 \wedge b_1 b_2 ... b_n \in L_2\}.$$

Prove or disprove: $L_1 \% L_2$ is regular.