Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
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Winter Semester 2014/15
Problem set 6
November 11, 2014

Automata and Formal Languages

Due November 18, 2014 before class!

Exercise 1 (Transducer - 10 points)

(a) Give a transducer over the alphabet $\Sigma = \{0, 1\}$ that recognizes
$L_1 = \{(a, b) \mid a, b \in \Sigma^* \wedge 2\text{lsbf}(a) = \text{lsbf}(b)\}$.

(b) By generalizing the transition function for transducer to $\delta : Q \times \Sigma^n \to Q$, we can construct automata the accept n-tuple of words.
Give an automaton over the alphabet $\Sigma = \{0, 1\}$ that recognizes
$L_2 = \{(a, b, c) \mid a, b, c \in \Sigma^* \wedge \text{lsbf}(a) + \text{lsbf}(b) = \text{lsbf}(c)\}$.

Exercise 2 (Transducer II - 10 points)

One of the limitations of transducers is that the accepted word-pairs have to be of the same length. To circumvent this limitation we introduce *ϵ-transducer*: They are defined similarly to usual transducers, however the transitions are labeled with $(\Sigma \cup \{\epsilon\} \times \Sigma \cup \{\epsilon\})$.

(a) Construct ϵ-transducers A_1 and A_2 such that $\mathcal{L}(A_1) = \{(a^n b^m, c^{2n}) \mid m, n \geq 0\}$ and $\mathcal{L}(A_2) = \{(a^n b^m, c^{2m}) \mid m, n \geq 0\}$.

(b) Compute the intersection of A_1 and A_2 using the algorithm for usual NFAs. What language does the resulting ϵ-transducer accept?

(c) Show that there is no ϵ-transducer that accepts $\mathcal{L}(A_1) \cap \mathcal{L}(A_2)$.

Exercise 3 (Transducer III - 10 points)

Show how to construct a transducer T over the alphabet $\Sigma \times \Sigma$ such that $(w, v) \in L(T)$ iff $wv \in L(A)$ and $|w| = |v|$.

Exercise 4 (Encoding - 10 points)

In the lecture we assumed, that every word $s \in \Sigma^*$ is encoded by all words $s\#^n$ for $n \geq 0$. This way words of different length can be paired up. In the projection- and join-algorithms we saw, that it is necessary to do a Pad-Closure after each operation. How do you have to change the Pad-Closure if instead of encoding s by all words $s\#^n$, s is encoded by all words $\#^n s$ for $n \geq 0$?