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Fall Semester
November 3, 2014

Randomized Algorithms

Exercise Sheet 4

Due: November 10, 2014

Exercise 1 (10 points)
Consider a uniform tree in which the root and every internal node have exactly 3 children. Every leaf is at distance h from the root (h is called the height of the tree) and it is associated with a boolean value 0 or 1. The value of a non-leaf node is the value of the majority of its children. The evaluation problem is to determine the value of the root.

- Show that for any deterministic algorithm A, there is an instance such that A will have to read all $n = 3^h$ leaves in order to evaluate the tree correctly.
- Consider the recursive randomized algorithm which evaluates two randomly selected sub-trees of the root and if their values disagree, then it evaluates the third sub-tree. Show that the expected number of leaves read by the algorithm is $O(n^{0.9})$.

Exercise 2 (10 points)
Consider a 2-player zero-sum game specified by the following payoff matrix:

$$M = \begin{pmatrix} 5 & 8 \\ 9 & 2 \end{pmatrix}$$

- Verify that there are no optimal pure strategies.
- Compute optimal mixed strategies for both players.
- What is the optimal strategy for each player assuming the he knows the mixed strategy of his opponent?

Exercise 3 (10 points)
Consider a 2-player zero-sum game specified by a $n \times m$ payoff matrix M.

- Show that $\max_i \min_j M_{i,j} \leq \min_j \max_i M_{i,j}$.
- If p and q denote mixed strategies for the row player and the column player, respectively, show that $\max_p \min_j \sum_i p_i M_{i,j} \leq \min_q \max_i \sum_j q_j M_{i,j}$.

Exercise 4 (10 points)
Consider the following problem. Given a string $x \in \{0,1\}^n$, we want to determine if x contains two consecutive 1. By using Yao's MinMax Principle, show that the expected number of bits inspected by any randomized algorithm is $\Omega(n)$.