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Summer Term 2015
Problem Set 05
May 11, 2015

Complexity Theory

Due date: May 18, 2015 **before** *class!*

Problem 1 (10 Points)

In the Exactly One 3Sat problem, we are given a 3CNF formula φ and need to decide if there exists a satisfying assignment u for φ such that every clause of φ has exactly one TRUE literal. Prove that Exactly One 3Sat is $\mathcal{NP}$-complete.

Problem 2 (10 Points)

Define the following two covering problems:

- A *vertex cover* of a graph $G = (V, E)$ is a set of vertices $V' \subseteq V$, where every edge in E is incident to at least one vertex in V'.
  Let Vertex Cover $= \{(G, k) : G$ has a vertex cover of size at most $k\}$.
- Given a set U, and a family S of subsets of U, a *set cover* of U is a subfamily of sets $C \subseteq S$ whose union is U.
  Let Set Cover $= \{(U, S, k) : U$ has a set cover of size at most $k\}$.

Show the following two claims.

1. Vertex Cover is $\mathcal{NP}$-complete.
2. Set Cover is $\mathcal{NP}$-complete.

Problem 3 (10 Points)

Define a *regular expression* r over $\{0, 1\}$ as

$$r ::= 0 \;\big|\; 1 \;\big|\; rr \;\big|\; (r|r),$$

or, equivalently,

$$\begin{aligned} r &\to 0 \\ r &\to 1 \\ r &\to rr \\ r &\to (r|r). \end{aligned}$$

The problem RegExpEq is about the question whether two languages defined by two different regular expressions are identical. A special case of this is the language $\textsc{RegExpEq}_*$, which is defined as

$$\textsc{RegExpEq}_* = \{r : \text{there exists an } n \in \mathbb{N} \text{ s.t. } L(r) = \Sigma^n\},$$

where $L(r)$ denotes the language generated by r, i.e., the set of all words that can be generated by using the rules of r.
Given $\Sigma = \{0, 1\}$, show that $\textsc{RegExpEq}_*$ is co$\mathcal{NP}$-complete.

Problem 4 (10 Points)

Define the class $\mathbf{DP} = \{L = L_1 \cap L_2 : L_1 \in \mathcal{NP}, L_2 \in \text{co}\mathcal{NP}\}$. (Note that we do not know if $\mathbf{DP} = \mathcal{NP} \cap \text{co}\mathcal{NP}$.) Consider the following languages:

$\textsc{ExactIndset} = \{(G, k) :$ the largest independent set of G has size exactly $k\}$,
$\textsc{Critical Sat} = \{\varphi : \varphi$ in 3CNF is unsatisfiable, but deleting any clause makes it satisfiable$\}$.

Show the following:

1. $\textsc{ExactIndset} \in \mathbf{DP}$.

2. $\textsc{Critical Sat}$ is $\mathbf{DP}$-complete.
   Hint: Use a $\mathbf{DP}$-complete problem and reduce it to $\textsc{Critical Sat}$. What would be the obvious choice for a $\mathbf{DP}$-complete problem?