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Online and Approximation Algorithms

Due June 8, 2015 before class!

Exercise 1 (Path Game - 10 points)

Consider the following 2-player game. There is a graph $G = (V, E)$ and the game takes place in alternating turns. In each turn, a player picks an edge $e \in E$ which has not been chosen by any player so that the selected edges form a single path. The first player who is unable to choose such an edge looses the game. Show that, if the starting player is given a perfect matching M of G, then there exists a winning strategy for him.

Exercise 2 (Randomized Matching - 10 points)

Consider the following randomized online algorithm for the maximum matching problem on bipartite graphs. Whenever a new vertex $v \in V$ arrives, match v with a vertex $u \in U$ chosen uniformly at random among the currently unmatched neighbors of v. Show that the competitive ratio of this algorithm cannot be better than $\frac{1}{2}$.

Hint: Consider a bipartite graph $G = (U \cup V, E)$ such that $U = \{u_1, u_2, \ldots, u_n\}$ and $V = \{v_1, v_2, \ldots, v_n\}$. The vertices u_i and v_j are connected iff either $1 \leq i, j \leq \frac{n}{2}$, or $i + j = n + 1$.