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Problem set 12
July 06, 2015
Summer Semester 2015

Online and Approximation Algorithms

Due July 13, 2015 before class!

Exercise 1 (Independent Set - 10 points)

In the independent set problem, we are given a connected undirected graph $G = (V, E)$ and we want to find a maximum subset of vertices which are pairwise non-adjacent. Propose a greedy algorithm for this problem and show that it achieves an approximation ratio of $\frac{1}{\Delta}$, where Δ is the maximum vertex degree.

Exercise 2 (MAX-NAE3SAT - 10 points)

We consider the *MAX-NAE3SAT* (Max-not-all-equal-3SAT) problem. In this problem we are given a *3CNF* formula and our goal is to find a variable assignment, such that the number of clauses containing at least one true **and** one false literal is maximized.
Develop a randomized approximation algorithm for *MAX-NAE3SAT* with an approximation factor of $\frac{3}{4}$.

Exercise 3 (2SAT - 10 points)

Consider the *2SAT* problem. In this problem we are given a boolean formula in *2CNF* and have to decide whether the formula is satisfiable or not.

(a) Describe a method of encoding the given *2CNF*-formula as a directed graph.

(b) Develop an algorithm that decides in polynomial time whether the given formula can be satisfied or not.

(c) Develop an algorithm that returns a satisfying assignment if it exists and show that it runs in polynomial time as well.

Exercise 4 (Randomized Max Cut - 10 points)

Let $G = (V, E)$ be an undirected graph with n vertices and m edges and assume that m is an even number. A *cut* C of G is a partition of the vertices in V into two disjoint sets A and B. The *size* of C is the number edges with one endpoint in A and the other in B. Consider the following randomized algorithm for finding a maximum cut. Each vertex $u \in V$ is assigned into one of the two sets A or B with probability $\frac{1}{2}$. Show that this algorithm achieves an approximation ratio of $\frac{1}{2}$. Then, derandomize the algorithm.