Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
Prof. Dr. Harald Räcke
Chintan Shah

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Efficient Algorithms and Datastructures I

Question 1 (10 Points)

An order-statistics tree is an augmented Binary Search Tree that supports the additional operations RANK(x), which returns the rank of x (i.e., the number of elements with keys less than or equal to x) and FINDBYRANK(k), which returns the kth smallest element of the tree.
Let $A[1, \cdots, n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an inversion of A. Show how to use an order-statistics tree to count the number of inversions in A in time $O(n \log n)$.

Question 2 (10 Points)

Show how to maintain a dynamic set Q of numbers that supports the operation MIN-GAP, which gives the magnitude of difference of the two closest numbers in Q. For example, if $Q = \{1, 5, 9, 15, 18, 22\}$, then MIN-GAP($Q$) returns 18-15=3, since 15 and 18 are the two closest numbers in Q. Make the operations INSERT, DELETE, SEARCH, and MIN-GAP as efficient as possible, and analyze their running times.

Question 3 (10 Points)

Suppose that we wish to keep track of a *point of maximum overlap* in a set of itervals - a point that has the largest number of intervals in the set of intervals overlapping it.

1. Show that there will always be a point of maximum overlap which is an endpoint of one of the segments.

2. Design a data structure that efficiently supports the operations INSERT, DELETE, and FIND_POM which are defined as follows:

   (a) INSERT(i, j): Inserts the interval $[i, j]$ in the set of intervals.

   (b) DELETE(i, j): Deletes the interval $[i, j]$ from the set of intervals.

   (c) FIND_POM: Returns a point of maximum overlap.

   (*Hint:* Keep a red-black tree of all the endpoints. Associate a value of $+1$ with each left endpoint, and associate a value of -1 with each right endpoint. Augment each node of the tree with some extra information to maintain the point of maximum overlap.)